Army Primary Standards Laboratory Uncertainty Analysis for a 50 GPM Positive Displacement Piston Prover

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Abstract

This paper is a detailed uncertainty analysis for a 50 gallon per minute positive displacement piston prover operated by the Liquid Flow Laboratory of the Army Primary Standards Laboratory (APSL). This uncertainty analysis encompasses all known contributors included in the APSL estimation of liquid flow piston prover uncertainty and includes consideration of connecting volume and viscosity. This uncertainty analysis is intended as an aid to those who work in the field of liquid flow metrology.

Introduction

A simple schematic of a positive displacement liquid flow piston prover is shown below in Figure 1.

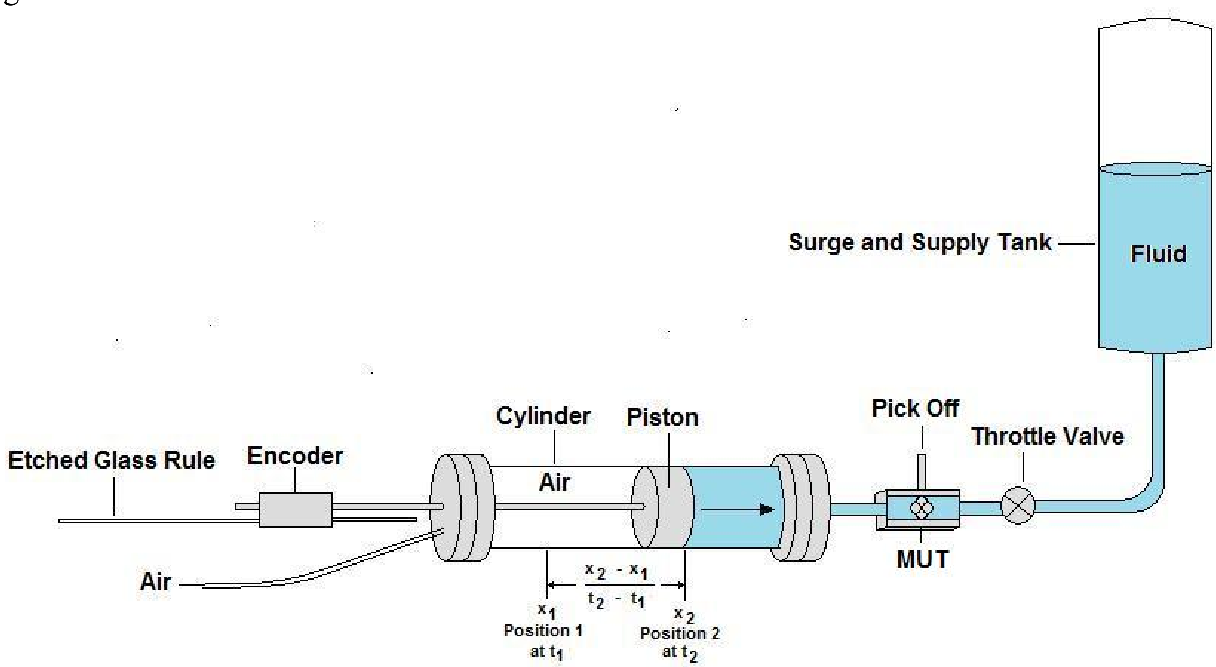


Figure 1. Simple Schematic of a Positive Displacement Piston Prover

The positive displacement liquid flow calibrator consists of a piston-cylinder flow element and measures flow rates through a Meter Under Test (MUT) as illustrated in the following manner. As shown in Figure 1, the piston starts at position 1, (x_1) at time (t_1) on the x axis, and being set into motion by air pressure, moves in a positive direction along the x axis to position 2, (x_2) in time (t_2). The displacement of the piston $(x_2 - x_1)$ divided by the time elapsed for the piston to move from position 1 to position 2 $(t_2 - t_1)$ gives us the rate [Unit Length/Unit Time] the piston has traveled in the positive x direction, and is given by the following expression.

$$Rate = \left(\frac{x_2 - x_1}{t_2 - t_1} \right) = \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{1}$$

Since the piston and cylinder are circular, the area is given by $A = \pi r^2$. If we multiply the constant area by the rate in Equation 1, the volumetric flow rate $\dot{v}$ of the positive displacement calibrator can be defined as.

$$\dot{v} = Area \times Rate = A\frac{dx}{dt} = \pi r^2 \frac{dx}{dt} = \frac{dv}{dt} \tag{2}$$

The distance, x, traveled by the piston in Figure 1 is measured by a graduated etched glass rule (with a typical grating pitch of 20 μm) that is read by an encoder which produces a pulse each time a graduation on the rule is encountered. The rate the piston travels in the x direction is influenced by the Throttle Valve. If the area of the piston-cylinder element is known, one can determine the volume flow as the area times the distance traveled.

Calibrator Constant, K_C [Calibrator Pulses/Unit Volume]

A dimensional calibration to determine the diameter and cylindricity of the flow tube is required to achieve traceability according to the above method. To avoid the many pitfalls associated with a three dimensional characterization, the APSL's Liquid Flow Laboratory performs volumetric fluid draws to achieve traceability [1]. The fluid draw involves collecting liquid from a discharge port on the piston prover in a calibrated flask of known volume and recording the number of pulses required to fill the flask to its etched graduation ring. The APSL performs eight separate draws along the length of the cylinder to calculate an average and obtain a repeatability/reproducibility figure. K_C is defined below.

$$K_C = \frac{P_C}{V} = \frac{[Calibrator\_Pulses]}{Unit\_Volume} \tag{3}$$

Meter K-Factor, K_{MUT} [Meter Pulses/Unit Volume]

Again referencing Figure 1, the equation used to determine the Meter K-Factor, K_{MUT} for a Turbine Flow Meter used as a MUT is:

$$K_{MUT} = \frac{P_{MUT}}{P_C} \times \frac{t_C}{t_{MUT}} \times K_{Translator} \quad (4)$$

Where

K_{MUT} = K-factor of the Turbine Meter Under Test .... [Meter Pulses/Unit Volume]
P_{MUT} = Output of the Meter ....[Pulses]
P_C = Number of Pulses Generated from the Calibrator's Encoder ....[Pulses]
t_C = Time over which the Calibrator's Pulses are collected.. [s]
t_{MUT} = Time over which the Turbine Meter's Pulses are collected .... [s]
$K_{Translator} = K_C \times (C_E \times C_T \times C_P)$

And

K_C = Calibrator Constant per Fluid Draw.................................. [Calibrator Pulses/Unit Volume]
C_E = Encoder Glass Thermal Expansion...[-]
C_T = Flow Tube Thermal Expansion ....[-]
C_P = Flow Tube Pressure Expansion ....[-]

Pulses Output of Meter Under Test (MUT), P_{MUT} [Pulses]

P_{MUT}, is the number of pulses output by the turbine meter during a measurement. The fluid is forced through the turbine meter and this flow causes the turbine meter blades to spin. The turbine meter MUT produces pulses from the rotation of the turbine blades, which are picked up by a detector and counted.

Calibrator Pulses, P_C [Pulses]

P_C is the number of pulses generated from the linear encoder on the calibrator reading the etched glass graduated ruler. Each pulse corresponds to a displacement along the x axis of 20 µm.

Calibrator Time, t_C [s]

In order to calculate the volumetric flow rate $\dot{v}$ we must have a time base to measure the duration over which pulses are counted, t_C. If we know how many pulses are collected we can relate the number of pulses to a unit volume and divide this by the calibrator time t_C to determine the volumetric flow rate $\dot{v}$. This is illustrated in equation 5 along with a dimensional analysis to prove out the units.

$$\dot{v} = \frac{1}{K_C} \times [P_C] \times \frac{1}{t_C} = \frac{Volume}{[Pulses]} \times [Pulses] \times \frac{1}{time} = \frac{Volume}{time} \qquad (5)$$

Meter Under Test Time, t_{MUT} [s]

t_{MUT} is the duration between the beginning of the collection of Meter Pulses to the end of the pulse collection. The time base used for measuring t_{MUT} is the same as the time base used for measuring t_C. Therefore, the uncertainty for drift is eliminated. When the MUT is a pulse producing device such as a turbine meter, a special timing technique referred to as Double Chronometry is used and is explained below.

Double Chronometry

Double Chronometry is a technique used in positive displacement provers/calibrators. It minimizes uncertainty by insuring that only whole undivided pulses are counted and timed for both the pulses produced by the MUT and the linear encoder used on the calibrator. This method eliminates the possibility of counting unknown fractions of pulses in the calibration time interval and eliminates uncertainties that could become potentially large, especially when using small volume displacement flow calibrators where pulse count is small for both the turbine meter and the linear encoder of the calibrator. The American Petroleum Institute (API) requires the minimum resolution of small volume provers be 1 part in 10,000 and this requires double chronometry [2].

The diagram below is provided to help illustrate how the double chronometry technique is applied.

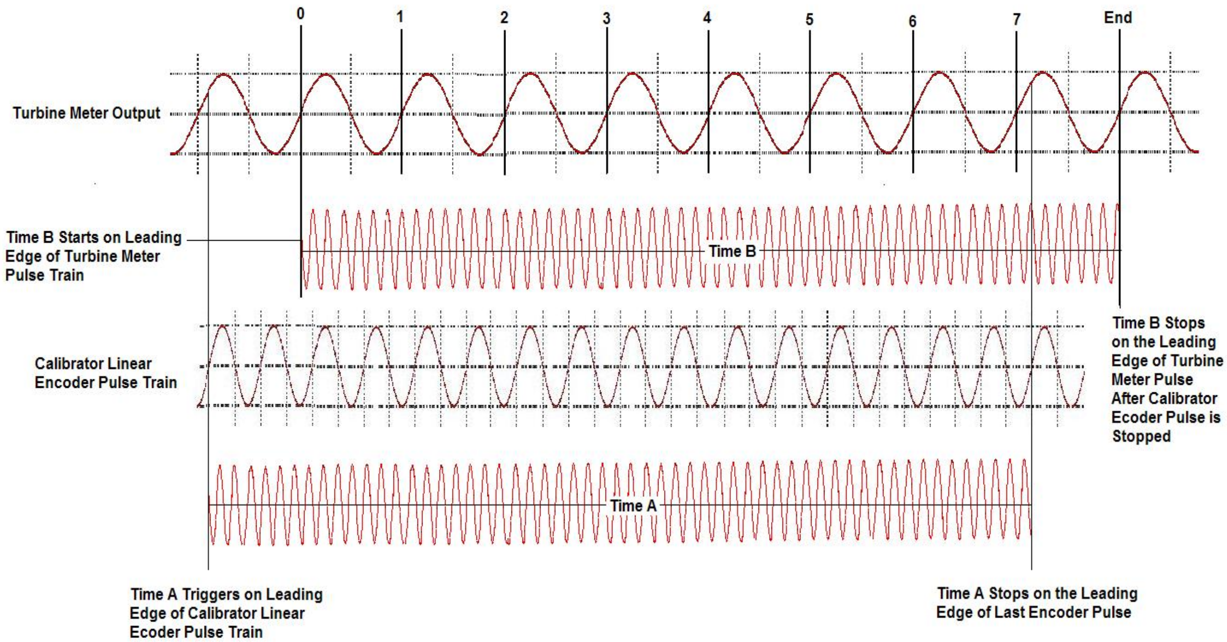


Figure 2. Double Chronometry

When air pressure is applied on the left side of the piston in Figure 1, the piston begins to move in the positive x direction and the Encoder starts producing pulses from reading the Graduated Glass Ruler. Timer A, in Figure 2, is started when the leading edge of the first encoder pulse is encountered. This event also enables the start gate of the Flow Meter Pulse Counter and Timer B. Timer B is started on the leading edge of the first pulse generated by the Turbine Meter. Timer A is stopped on the leading edge of the last encoder pulse and Timer B is stopped on the leading edge of the last Flow Meter pulse detected. In this way only full pulses are counted.

Referring to Figure 2, where Time A = t_C and Time B = t_{MUT}

Flow Meter Frequency, f_{MUT}

$$f_{MUT} = \frac{Meter\_Pulses}{Meter\_Time} = \frac{P_{MUT}}{t_{MUT}} \tag{6}$$

Volumetric Flow Rate, $\dot{v}$

$$\dot{v} = \frac{Encoder\_Pulses}{(Encoder\_Time \times Calibrator\_K\_Factor)} = \frac{P_C}{(t_C \times K_C)} \tag{7}$$

Mass Flow Rate, $\dot{m}$

$$\dot{m} = \left(Encoder\_Pulses\right) \times \left(\frac{Fluid\_Density}{\left(Encoder\_Time\right) \times \left(Calibrator\_K\_Factor\right)} \right) = P_C \times \left(\frac{\rho_{fluid}}{t_C \times K_C} \right) \tag{8}$$

Meter K-factor, K_{MUT}

$$K_{MUT} = \frac{Flow\_Meter\_Frequency}{Flow\_Rate} = \frac{\left(\dfrac{P_{MUT}}{t_{MUT}}\right)}{\left(\dfrac{P_C}{t_C \times K_C}\right)} \tag{9}$$

The Meter K-factor, K_{MUT}, is shown in Equation (9) and in Equation (4). (The apparent difference between Equation (9) and Equation (4) is due to the term $K_{translator}$ which, if substituted for K_C in Equation (9) gives us Equation (4))

$$K_{MUT} = \frac{\left(\dfrac{P_{MUTr}}{t_{MUT}}\right)}{\left(\dfrac{P_C}{t_C \times K_C}\right)} = \left(\frac{P_{MUT}}{t_{MUT}}\right) \times \left(\frac{t_C \times K_C}{P_C}\right) = \frac{P_{MUT}}{P_C} \times \frac{t_C}{t_{MUT}} \times K_C \tag{10}$$

Substituting $K_{translator}$ for K_C

$$K_{MUT} = \frac{P_{MUT}}{P_C} \times \frac{t_C}{t_{MUT}} \times K_{Translator} \tag{11}$$

$K_{translator}$ is a variable used to correct for the Thermal Expansion of the Glass Ruler on the Encoder, the Thermal Expansion of the Flow Tube and the Pressure Expansion of the Flow Tube. The next section will cover the variable $K_{translator}$.

Translator Corrections $K_{translator}$, [Pulses/Unit Volume]

$K_{translator}$ corrects K_C for Thermal and Pressure Expansion of the Flow Tube as well as the thermal expansion of the glass ruler of the encoder used on the calibrator and is defined as

$$K_{Translator} = K_C \times \left(C_E \times C_T \times C_P\right) \tag{12}$$

Where the calibrator constant K_C, is the calibrator constant defined above.

Encoder Glass Thermal Expansion, C_E **[-]**

The equation for correcting for the thermal expansion of the encoder is given below.

$$C_E = \left[1 - \alpha_{ENC}\left(T_{AMB} - T_{REF}\right)\right] \tag{13}$$

Where

α_E = The Linear Thermal Expansion Coefficient of the Encoder.................................. [/°F or °C]
T_{AMB} = Ambient Temperature .... [°F or °C]
T_{REF} = Reference Temperature .... [°F or °C]

Thermal Expansion of Flow Tube C_T **[-]**

The equation for correcting for the thermal expansion of the flow tube is

$$C_T = \left[1 - \alpha_T\left(\overline{T}_{STD} - T_{REF}\right)\right] \tag{14}$$

Where

α_T = The Linear Thermal Expansion Coefficient of the Flow Tube.............................. [/°F or °C]
$\overline{T}_{STD}$ = Temperature of the Fluid .... [°F or °C]
T_{REF} = Reference Temperature .... [°F or °C]

Pressure Expansion of Flow Tube C_P **[-]**

$$C_P = \left[1 - \alpha_P\left(P_{STD} - P_{REF}\right)\right] \tag{15}$$

Where

α_P = The Pressure Expansion Coefficient of the Flow Tube .... [/Unit Pressure]
P_{STD} = Pressure of the Fluid .... [Unit Pressure]
P_{REF} = Reference Pressure .... [Unit Pressure]

Meter K-Factor Equation with All Corrections

By applying the above corrections to equation 11, the following relationship is obtained. This equation is used by the APSL Liquid Flow Laboratory for this particular 50 gpm piston prover which has only one temperature probe and it is assumed that calibrator and meter temperature are the same. APSL flow calibration stands that have multiple probes permit direct accounting for liquid density differences at the calibrator and at the meter.

$$K_{MUT} = \frac{P_{MUT}}{P_C} \times \frac{t_C}{t_{MUT}} \times K_C \cdot \left(1 - \alpha_{ENC} \cdot \left(T_{AMB} - T_{REF}\right)\right) \cdot \left(1 - \alpha_T \cdot \left(\overline{T}_{STD} - T_{REF}\right)\right) \cdot \left(1 - \alpha_P \cdot \left(P_{STD} - P_{REF}\right)\right) \quad (16)$$

Connecting Volume

In a paper entitled, "Accounting for the Impact of Thermal Instability in the Liquid Comprising the Connecting Volume of a Piston Displacement type Volumetric Flow Rate Standard", by Jeremy Latsko (AFMETCAL) and James Winchester (Arnold Engineering Development Center, Arnold Air Force Base, Tennessee) [3], the authors quantify the thermal instabilities in the connecting volume of a prover where the connecting volume is defined as the fluid residing in the volume between the metering piston and the meter under test during standard volume delivery. The authors start with the conservation of mass to derive the following equation assuming the density of the liquid is different at the discharge port of the piston-cylinder element and the MUT and a term is added to account for connecting volume:

$$\overline{\dot{V}}_{MUT} = \overline{\dot{V}}_{STD} \cdot \left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right) + \frac{V_{CV}}{V_{STD}} \cdot \beta \cdot \left(\langle T_{CVf} \rangle - \langle T_{CVi} \rangle\right)\right] \quad (17)$$

The details of the derivation of equation 17 can be seen in Section A of the Appendices.

Where:

$\overline{\dot{V}}_{MUT} =$ The time average volumetric flow rate at the meter under test during delivery of the standard volume.

$\overline{\dot{V}}_{STD} =$ The time average volumetric flow rate at the piston during delivery of the standard volume.

$\beta =$ The volumetric thermal expansion coefficient of the fluid in the calibration system at the nominal operating temperature at constant pressure.

$$\beta = \frac{1}{V} \cdot \frac{\partial V}{\partial T} = -\frac{1}{\rho} \cdot \frac{\partial \rho}{\partial T}$$

$\overline{T}_{MUT} =$ The time average temperature at the meter under test during the delivery of the standard volume.

$\overline{T}_{STD} =$ Time average temperature at the piston during the delivery of the standard volume.

$V_{CV} =$ The connecting volume between the meter under test and the piston, the sum of the piping volume from the cylinder discharge port to the meter under test and the volume in the cylinder between the cylinder discharge port and the piston at the end of standard volume delivery.

V_{STD} = The standard volume derived from piston displacement in the cylinder of known cross section.

$\langle T_{CVf} \rangle$ = The spatial average fluid temperature in the connecting volume at the end of standard volume delivery. Temperature units depend on those of the β being used.

$\langle T_{CVi} \rangle$ = The spatial average fluid temperature in the connecting volume at the initiation of standard volume delivery. Temperature units depend on those of the β being used.

The term $\beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right)$ is zero because when $\overline{T}_{MUT} = \overline{T}_{STD}$, as is the case for the APSL 50 gpm prover addressed in this paper. However, a contribution for this term will be applied in the uncertainty analysis.

Combining Connecting Volume with the APSL Liquid Flow Mathematical Model

K_C is related to volumetric flow rate using the following relationship:

$$\overline{\dot{V}}_{STD} = \frac{1}{K_C} \cdot P_C \cdot \frac{1}{t_C} \Rightarrow K_C = \frac{1}{\overline{\dot{V}}_{STD}} \cdot P_C \cdot \frac{1}{t_C} \tag{18}$$

Meter K_{MUT} is related to volumetric flow rate using the following relationship:

$$\overline{\dot{V}}_{MUT} = \frac{1}{K_{MUT}} \cdot P_{MUT} \cdot \frac{1}{t_{MUT}} \Rightarrow K_{MUT} = \frac{1}{\overline{\dot{V}}_{MUT}} \cdot P_{MUT} \cdot \frac{1}{t_{MUT}} \tag{19}$$

Inserting $\overline{\dot{V}}_{STD}$ and $\overline{\dot{V}}_{MUT}$ from equations 18 and 19 into equation 17 the following relationship is obtained in terms of K_C and K_{MUT}.

$$\frac{P_{MUT}}{K_{MUT} \cdot t_{MUT}} = \frac{P_C}{K_C \cdot t_C} \cdot \left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right) + \frac{V_{CV}}{V_{STD}} \cdot \beta \cdot \left(\langle T_{CVf} \rangle - \langle T_{CVi} \rangle\right)\right] \tag{20}$$

Rearranging and solving for K_{MUT}

$$\frac{P_{MUT}}{K_{MUT} \cdot t_{MUT}} = \frac{P_C}{K_C \cdot t_C} \cdot \left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right) + \frac{V_{CV}}{V_{STD}} \cdot \beta \cdot \left(\langle T_{CVf} \rangle - \langle T_{CVi} \rangle\right)\right] \tag{21}$$

$$\frac{1}{K_{MUT}} = \frac{P_C \cdot t_{MUT}}{P_{MUT} \cdot K_C \cdot t_C} \cdot \left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right) + \frac{V_{CV}}{V_{STD}} \cdot \beta \cdot \left(\langle T_{CVf} \rangle - \langle T_{CVi} \rangle\right)\right] \tag{22}$$

$$K_{MUT} = \frac{P_{MUT}}{P_C} \cdot \frac{t_C}{t_{MUT}} \cdot K_C \cdot \frac{1}{\left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right) + \frac{V_{CV}}{V_{STD}} \cdot \beta \cdot \left(\left\langle T_{CVf}\right\rangle - \left\langle T_{CVi}\right\rangle\right)\right]} \tag{23}$$

Inserting $K_{Translator}$ correction

$$K_{Translator} = K_C \times \left(C_E \times C_T \times C_P\right) \tag{24}$$

Into equation 23 the following relationship is obtained.

$$K_{MUT} = \frac{P_{MUT}}{P_C} \cdot \frac{t_C}{t_{MUT}} \cdot K_C \cdot \frac{\left(C_E \times C_T \times C_P\right)}{\left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right) + \frac{V_{CV}}{V_{STD}} \cdot \beta \cdot \left(\left\langle T_{CVf}\right\rangle - \left\langle T_{CVi}\right\rangle\right)\right]} \tag{25}$$

Where

$C_E = \left[1 - \alpha_{ENC}\left(T_{AMB} - T_{REF}\right)\right]$ is the thermal expansion of the encoder (26)

$C_T = \left[1 - \alpha_T\left(\overline{T}_{STD} - T_{REF}\right)\right]$ is the thermal expansion of the flow tube (27)

$C_P = \left[1 - \alpha_P\left(P_{STD} - P_{REF}\right)\right]$ is the pressure expansion of the flow tube (28)

Including the corrections in equation 25 the following equation is obtained

$$K_{MUT} = \frac{P_{MUT}}{P_C} \cdot \frac{t_C}{t_{MUT}} \cdot K_C \cdot \frac{\left[1 - \alpha_{ENC}\left(T_{AMB} - T_{REF}\right)\right] \cdot \left[1 - \alpha_T\left(\overline{T}_{STD} - T_{REF}\right)\right] \cdot \left[1 - \alpha_P\left(P_{Fluid} - P_{REF}\right)\right]}{\left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right) + \frac{V_{CV}}{V_{STD}} \cdot \beta \cdot \left(\left\langle T_{CVf}\right\rangle - \left\langle T_{CVi}\right\rangle\right)\right]} \tag{29}$$

Equation 29 is the mathematical model for the APSL 50 gpm piston prover. It includes terms for the temperature densities at the calibrator and the MUT and also the connecting volume applied to the APSL 50 gpm piston prover.

Uncertainty Analysis

To begin the uncertainty analysis the sensitivity coefficients for each variable in equation 29 will be calculated using partial differentiation. The sensitivity coefficients mathematically describe how an uncertainty in K_{MUT} would be influenced by changes in the quantity or variable of interest in equation 29. The calculated sensitivity coefficient for each influence quantity in equation 29 is shown in Section B of the Appendices.

Calibrator Pulses, P_C [Calibrator Pulses]

The uncertainty of the encoder pulse counts is determined by the uncertainty of the pulse spacing generated by the linear encoder; there is no uncertainty due to the counter because the counter simply counts pulses (it either sees a pulse or it does not).

The Mitutoyo encoder used in this particular Prover is a model AT102 which has a grating pitch of 20 μm/pulse. A total of 2741 pulses were collected during this fluid draw. Therefore, the piston traveled $(2741 pulses)(20 \mu m) = 54.82 mm$. The Spec Sheet for the Mitutoyo Model AT102-500 Code No. 539-119 Linear Scale Unit Manual No. 4739GB Series No. 539 [4] provides the uncertainty for the Model AT102-500 Code 539-119 as $U = (5 + 5L / 10000 \mu m)$ where L is the distance traveled by the encoder which in this case is 54.82 mm. Then $U = \pm(5 + 5 \cdot (54.82 / 10000)) \times 10^{-3} mm = \pm 0.005027 mm$. With a grating pitch of 20 μm/pulse this can be converted into

$(0.005027 \times 10^{-3} m)/(20 \times 10^{-6} m / pulse) = 0.2514 pulses$.

The total number of pulses collected during a water draw was 2741. Thus converting to percent error we obtain the following.

$$\frac{(0.2541 pulses)}{(2741 pulses)} \cdot 100\% = 0.0092\%$$

Uncertainty Summary:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial P_C} = -P_C^{-1}$

Distribution: Normal

One Standard Uncertainty: $\frac{(0.2541 pulses)}{2} \cdot = 0.1257 pulses$

Calibrator Time, t_C [Unit Time]

The duration over which a whole number of calibrator pulses are collected is measured by the calibrator clock. The clock used by this 50 gpm flow calibrator is supplied by a Measurement Computing PCI-CTR10 card. From the Measurement Computing PCI-CTR10 Specification Sheet Document Revision 1.2 dated June 2006 [5] the accuracy of the frequency of the clock is 50 ppm. The uncertainty summary for the calibrator time is shown below.

Uncertainty Summary:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial t_C} = t_C^{-1}$

Distribution: Rectangular

One Standard Uncertainty**:** $\frac{(1s)\cdot(50\times10^{-6})}{\sqrt{3}} = 2.886751\times10^{-5}\,s$

Meter Under Test Pulses, P_{MUT} [MUT Pulses]

There is no uncertainty due to 50 gpm calibrator's counter because the counter simply counts pulses. Assuming that there is no interference in the RF pickoff or interruption in the spinning blades of the Turbine Meter, the uncertainty associated with the counted flow meter pulses is zero.

Uncertainty Summary:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial P_{MUT}} = P_{MUT}^{-1}$

Distribution: Rectangular

One Standard Uncertainty**:** $\frac{0}{\sqrt{3}} = 0\,pulses$

Meter Under Test Time, t_{MUT} [Unit Time]

Similarly, the duration over which a whole number of calibrator pulses are collected is measured by the calibrator clock. The uncertainty summary for the calibrator time is shown below.

Uncertainty Summary:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial t_{MUT}} = -t_{MUT}^{-1}$

Distribution: Rectangular

One Standard Uncertainty: $\dfrac{(1s)\cdot(50\times10^{-6})}{\sqrt{3}} = 2.886751\times10^{-5}\,s$

Calibrator Constant, K_C [Calibrator Pulses/Unit Volume]

Traceability for the APSL piston prover is achieved through the volumetric fluid draw technique where a known traceable volume is used to determine how many calibrator pulses are required to fill it [1]. It is through the fluid draw that the calibrator constant is derived. The Fluid Draw Uncertainty for this 50 gpm prover is shown in Section C of the Appendices.

The prover K-factor K_C from this water draw was determined to be 2740.8099 pulses per liter with an overall expanded uncertainty of 0.04160%.

Uncertainty Summary:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $\dfrac{1}{K_{MUT}}\cdot\dfrac{\partial K_{MUT}}{\partial K_C} = K_C^{-1}$

Distribution: Normal

One Standard Uncertainty**:** $\dfrac{2740.8099\,pulses\cdot\left(0.04160\%/100\%\right)}{2} = 0.57008846\,pulses$

Encoder Thermal Expansion Coefficient, α_{ENC} [°F^{-1}]

This 50 gpm stand at the APSL utilizes a Mitutoyo Model AT102 Encoder that, according to the Mitutoyo online catalog specifications, [6] has a thermal expansion coefficient of $(8\pm1)\times10^{-6}{}^\circ C^{-1}$ with an uncertainty of $\pm1\times10^{-6}{}^\circ C^{-1}$ which converts into $(5/9)^\circ C/^\circ F\cdot 8\times10^{-6}/^\circ C = 4.44\times10^{-6}/^\circ F$ with an uncertainty of $(5/9)^\circ C/^\circ F\cdot\pm1\times10^{-6}/^\circ C = 5.56\times10^{-7}/^\circ F$.

Uncertainty Summary:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $\dfrac{1}{K_{MUT}}\cdot\dfrac{\partial K_{MUT}}{\partial\alpha_{ENC}} = \dfrac{-(T_{AMB}-T_{REF})}{[1-\alpha_{ENC}\cdot(T_{AMB}+T_{REF})]}$

Distribution: Rectangular

$$\text{One Standard Uncertainty: } \frac{5.56\times10^{-6}{}^\circ F^{-1}}{\sqrt{3}} = 3.21\times10^{-7}{}^\circ F^{-1}$$

Ambient Temperature Measurement, T_{AMB} [°F]

The APSL's Flow Laboratory is kept at 20°C ±2°C or 68°F ±3.6 °F. Assuming the laboratory is fluctuating around 68°F by 3.6°F, the following uncertainty summary is derived for the ambient temperature measurement.

Uncertainty Summary:

Type of Uncertainty: Relative Type B

$$\text{Sensitivity Coefficient: } \frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial T_{AMB}} = \frac{-\alpha_{ENC}}{\left[1-\alpha_{ENC}\cdot\left(T_{AMB}+T_{REF}\right)\right]}$$

Distribution: Rectangular

$$\text{One Standard Uncertainty: } \frac{3.6^\circ F}{\sqrt{3}} = 2.078461^\circ F$$

Thermal Expansion Coefficient of Flow Tube, α_T [°F^{-1}]

The Flow Tube is constructed from 316 Stainless Steel, which according to API Manual of Petroleum Measurement Standards Chapter 12-Calculation of Petroleum Quantities, page 16, Table 6 [7], has a linear thermal expansion coefficient of 8.83x10^{-6} °F^{-1}. Since we want an area thermal expansion coefficient this value must be multiplied by 2 therefore the area thermal expansion coefficient of the stainless steel flow tube is 2(8.83x10^{-6}) =1.766x10^{-5} °F^{-1}. The online NIST Engineering Metrology Tool Box (http://emtoolbox.nist.gov/Temperature/Slide14.asp) [8] states that the value for the linear thermal expansion of 316 stainless steel is known within 3 to 5%. Therefore taking 5% as the worst case the following is obtained for the uncertainty in the volume area thermal expansion coefficient.

$$U = \sqrt{\left(\left(5/100\right)\cdot\left(8.83\times10^{-6}{}^\circ F^{-1}\right)\right)^2 + \left(\left(5/100\right)\cdot\left(8.83\times10^{-6}{}^\circ F^{-1}\right)\right)^2} = 6.2438\times10^{-7}{}^\circ F^{-1}$$

Uncertainty Summary:

Type of Uncertainty: Relative Type B

$$\text{Sensitivity Coefficient: } \frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial \alpha_T} = \frac{-\left(T_{STD}-T_{REF}\right)}{\left[1-\alpha_T\cdot\left(T_{STD}-T_{REF}\right)\right]}$$

Distribution: Rectangular

One Standard Uncertainty: $\dfrac{6.2438\times10^{-7}\circ F^{-1}}{\sqrt{3}} = 3.605\times10^{-7}\circ F^{-1}$

Average Temperature of Calibrator Fluid Media, $\overline{T}_{STD}$ [°F]

The Marlin PRT installed in this 50 gpm stand has a tolerance of 0.1% as provided by the online product catalog [9]. The uncertainty for the calibrator fluid temperature measurement is given below in the uncertainty summary.

Uncertainty Summary:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient:

$$\frac{1}{K_{MUT}}\cdot\frac{\partial K_{MUT}}{\partial \overline{T}_{STD}} = \frac{\alpha_T\cdot\left[V_{STD}+V_{STD}\cdot\beta\cdot\left(\overline{T}_{MUT}-T_{REF}\right)+V_{CV}\cdot\beta\cdot\left(\langle t_{CVf}\rangle-\langle t_{CVi}\rangle\right)\right]-V_{STD}\cdot\beta}{\left[-1+\alpha_T\cdot\left(\overline{T}_{STD}-T_{REF}\right)\right]\cdot\left[V_{STD}+V_{STD}\cdot\beta\cdot\left(\overline{T}_{STD}-T_{REF}\right)+V_{CV}\cdot\beta\cdot\left(\langle t_{CVf}\rangle-\langle t_{CVi}\rangle\right)\right]}$$

Distribution: Rectangular

One Standard Uncertainty: $\dfrac{68°F\cdot\left(0.1\%/100\right)}{\sqrt{3}} = 0.039259818°\text{F}$

Reference Temperature, T_{REF} [°F^{-1}]

The reference temperature is an assumed constant and exact. Therefore, there is not an uncertainty associated with reference temperature.

Uncertainty Summary

Type of Uncertainty: Relative Type B

Sensitivity Coefficient:

$$\frac{1}{K_{MUT}}\cdot\frac{\partial K_{MUT}}{\partial T_{REF}} = \frac{\alpha_{ENC}\cdot\left[1+\alpha_T\cdot\left(\overline{T}_{STD}-T_{REF}\right)\right]+\alpha_T\cdot\left[1+\alpha_{ENC}\cdot\left(T_{AMB}-T_{REF}\right)\right]}{\left[1+\alpha_T\cdot\left(\overline{T}_{STD}-T_{REF}\right)\right]\cdot\left[1+\alpha_{ENC}\cdot\left(T_{AMB}-T_{REF}\right)\right]}$$

Distribution: Rectangular

One Standard Uncertainty: 0

Pressure Expansion Coefficient of Flow Tube, α_P [psig^{-1}]

From the API MPM Ch12.2.1-1995 (R2009) Appendix A-6 page 19 [10], equation 30 was obtained.

$$C_{PS} = 1 + \left(\frac{P \times ID}{\gamma \times WT} \right) \tag{30}$$

Where:

C_{ps} = Correction for the effect of pressure on steel.
P = Pressure
ID = Tube Inside Diameter
γ = Modulus of Elasticity
WT = Tube Wall Thickness

The details for the calculations of Pressure Expansion Coefficient for the flow tube are available for review in Section D of the Appendices. The uncertainty summary for the Pressure Expansion Coefficient of the Flow Tube follows from the calculations in the Appendices.

Uncertainty Summary:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial \alpha_P} = \frac{-\left(P_{STD} - P_{REF}\right)}{\left[1 - \alpha_P \cdot \left(P_{STD} + P_{REF}\right)\right]}$

Distribution: Normal

One Standard Uncertainty: $\frac{\left(1.14286 \times 10^{-6}\right)\left(0.0005803\% / 100\%\right)}{\sqrt{3}} = 3.8289966 \times 10^{-12}\ psig^{-1}$

Fluid Pressure Measurement, P_{STD} [psig]

The run pressure gauge, which is read directly off the 50 gpm prover is an ASME Grade B (±3-2-3% of Span) gauge [11]. Typically the run pressure is set at approximately 60 psig. Since 60 psig is in the middle 50% of the gauge span the uncertainty is ±2% of span. The uncertainty summary follows.

Uncertainty Summary:

Type of Uncertainty: Relative Type B

$$\text{Sensitivity Coefficient: } \frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial P_{STD}} = \frac{-\alpha_P}{[1-\alpha_P \cdot (P_{STD} + P_{REF})]}$$

Distribution: Rectangular

$$\text{One Standard Uncertainty: } \frac{((2/100) \cdot 160\,psi)}{\sqrt{3}} \cdot = 1.85\,psig$$

Reference Pressure, P_{REF} **[psig]**

The reference pressure is assumed to be 0 psi gauge pressure. This value is arbitrary and is defined to be exact. Therefore this no uncertainty associated with this value.

Uncertainty Summary

Type of Uncertainty: Relative Type B

$$\text{Sensitivity Coefficient: } \frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial P_{REF}} = \frac{-\alpha_P}{[-1+\alpha_P \cdot (P_{STD} - P_{REF})]}$$

Distribution: Rectangular

One Standard Uncertainty: 0

Thermal Expansion Coefficient of the Liquid Media, β **[°F^{-1}]**

This 50 gpm prover is equipped with only one temperature probe that is located approximately 7 inches from the delivery port of the piston cylinder element and approximately 13 inches from the MUT. The data acquisition software is set to $T_{MUT} = T_{STD}$ making the $\beta \cdot (\overline{T}_{MUT} - T_{STD}) = 0$ in equation 29. However, we will still consider this term to make a point in our uncertainty.

The Liquid Media in the 50 gpm stand is MIL-PRF-7024E Type II blended with Pennant 460 gear oil. The thermal expansion coefficient for this fluid was determined by making several density measurements at several different temperatures using an Anton Paar DMA 5000 M density meter. The data and the calculations for the Thermal Expansion Coefficient of the Liquid Media are shown in Section E of the Appendices. The uncertainty summary for the Thermal Expansion Coefficient of the Liquid Media is shown below.

Uncertainty Summary for thermal expansion of MIL-PRF-7024 E Type II and Pennant 460 Gear Oil Blend:

Type of Uncertainty: Relative Type B

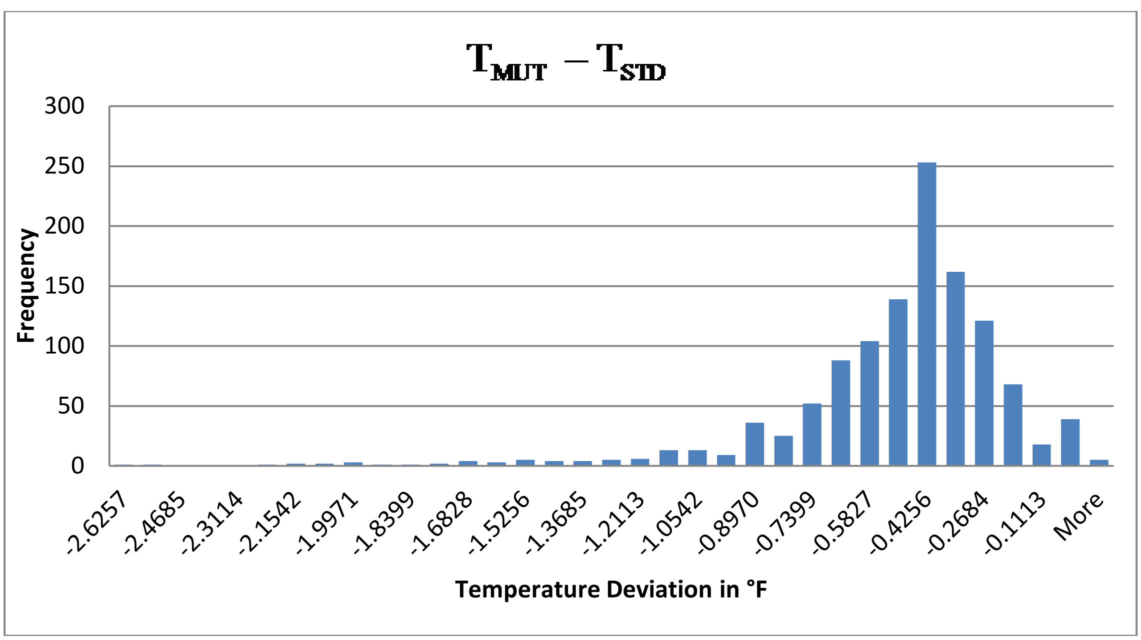
Sensitivity Coefficient: $\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial \beta} = -\frac{V_{STD} \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right) + V_{CV} \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)}{V_{STD} \cdot \left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right)\right] + V_{CV} \cdot \beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)}$

Distribution: Normal

One Standard Uncertainty: $\frac{(5.686040\% / 100) \cdot \left(4.90069 \times 10^{-4} {}^{\circ}F^{-1}\right)}{2} = 1.393276 \times 10^{-5} {}^{\circ}F^{-1}$

Temperature Deviation between Meter Temperature and Calibrator Temperature, $\Delta\overline{T} = \overline{T}_{MUT} - \overline{T}_{STD}$ [°F]

This 50 gpm piston prover utilizes one temperature probe inserted in 1 inch diameter (0.86 inch id) line approximately 7.5 inches from the discharge port of the piston cylinder element and approximately 13 inches from the meter. Since there is only one temperature probe the data acquisition software is set to Master Temperature = Meter Temperature or $\overline{T}_{MUT} = \overline{T}_{STD}$. In an effort to come up with a worst case estimation reference will be made to a 10 gpm bidirectional piston prover that has been modified for unidirectional operation. The reason for referring to this stand is that it has a Meter Temperature probe T_{MUT} and a Calibrator Temperature probe T_{STD} that are inserted into 1 inch diameter tubing, spaced approximately 86.5 inches apart. This stand also has temperature control and calibrates meters at a much lower flow rate than the 50 gpm stand. All these above characteristics induce a larger $\Delta\overline{T} = \overline{T}_{MUT} - \overline{T}_{STD}$ than the 50 gpm stand and make a hypothetical worst case scenario. The $\Delta\overline{T}$ taken from 1190 measurements of 17 different FT0-4 meters was averaged and an average $\Delta\overline{T}$ of 0.55 °F was calculated. See the Distribution of Temperature Deviations in Graph 1 below for the 10 gpm stand.



Graph 1. Distribution of Temperature Deviations between $\overline{T}_{MUT}$ and $\overline{T}_{STD}$ on a 10 gpm piston prover.

Average Temperature of Fluid in the Meter Under Test, $\overline{T}_{MUT}$ [°F]

The Marlin PRT installed in the 50 gpm piston prover has a tolerance of 0.1% as provided by the online product catalog [9]. The uncertainty for the fluid temperature measurement is given below in the uncertainty summary.

Uncertainty Summary:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $$\frac{1}{K}\cdot\frac{\partial K}{\partial \overline{T}_{MUT}} = \frac{-\beta\cdot V_{STD}}{V_{STD}\cdot\left[1+\beta\cdot\left(\overline{T}_{MUT}-\overline{T}_{STD}\right)\right]+V_{CV}\cdot\beta\cdot\left(\langle t_{CVf}\rangle-\langle t_{CVi}\rangle\right)}$$

Distribution: Rectangular

One Standard Uncertainty: $$\frac{68°F\cdot\left(0.1\% / 100\right)}{\sqrt{3}} = 0.039259818°\text{F}$$

Connecting Volume Uncertainty Components

This 50 gpm stand is not outfitted for all the measurements necessary to compensate for connecting volume. However, we will consider all the components of connecting volume in this uncertainty and make some educated assertions based on APSL data and experience to evaluate the effect of connecting volume in the evaluation of uncertainty on this particular piston prover.

Connecting Volume, V_{CV} [in³]

The Connecting Volume for this 50 gpm stand consists of 25 inches of 1 inch (0.86 id) tubing which runs between the delivery port of the piston cylinder element and the MUT plus an additional 6 in^3 of fluid volume remaining in the cylinder when the piston is at the end of standard volume delivery. Therefore, the connecting volume is

$$\frac{(25\cdot in)\pi(0.86\cdot in)^2}{4} + 6.00\cdot in^3 = 20.522\cdot in^3$$

It is estimated by the Liquid Flow Laboratory of the APSL that this connecting volume measurement is accurate to within 12%. The uncertainty summary for connecting volume follows

Uncertainty Summary

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial V_{CV}} = \frac{\beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)}{-\left[V_{STD} \cdot \left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right)\right] + V_{CV} \cdot \beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)\right]}$$

Distribution: Rectangular

One Standard Uncertainty: $$\frac{20.522° in^3 \cdot \left(12\% / 100\right)}{2} = 1.23132 \cdot \mathrm{in}^3$$

Volume of Standard, V_{STD} [in^3]

Latsko and Winchester define the Volume of the Standard V_{STD} to be the volume derived from piston displacement in the cylinder of known cross section. From this definition it is obvious that as the piston travels down the cylinder, the volume V_{STD} will decrease and as V_{STD} decreases the ratio of $\frac{V_{CV}}{V_{STD}}$ in the sensitivity coefficient will increase making the contribution of the connecting volume more and more significant as demonstrated in the equation below.

$$LIM_{V_{STD} \to 0} = \frac{V_{CV}}{V_{STD}} \cdot \frac{\beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)}{V_{STD} \cdot \left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right)\right] + V_{CV} \cdot \beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)} = \infty$$

Furthermore, the sensitivity coefficients of $\overline{T}_{STD}$, β, $\overline{T}_{MUT}$, V_{CV}, $\langle t_{CVf} \rangle$ and $\langle t_{CVi} \rangle$ will be influenced by changes in V_{STD}. Therefore, to test the effect that V_{STD} had on our overall uncertainty we created spread sheet and we used the following relationship where the effective diameter of the piston cylinder is 6.00021573487587 inches, the flow tube is 20 inches long, and Δx is the piston displacement.

$$V_{STD} = \pi \cdot \left(\frac{6.00021573487587 \cdot \mathrm{in}}{2}\right)^2 \cdot \left(20 \cdot in - \Delta x \cdot in\right)$$

For this 50 gpm prover we observed as we simulated moving the piston down the tube using the spreadsheet the effect piston displacement had on the overall uncertainty contribution

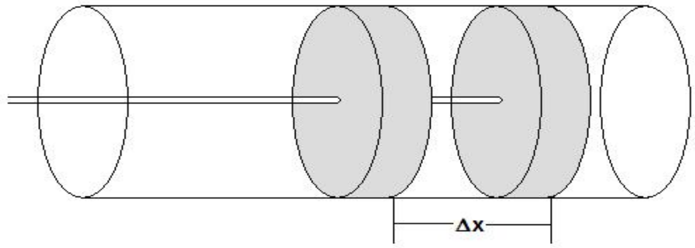
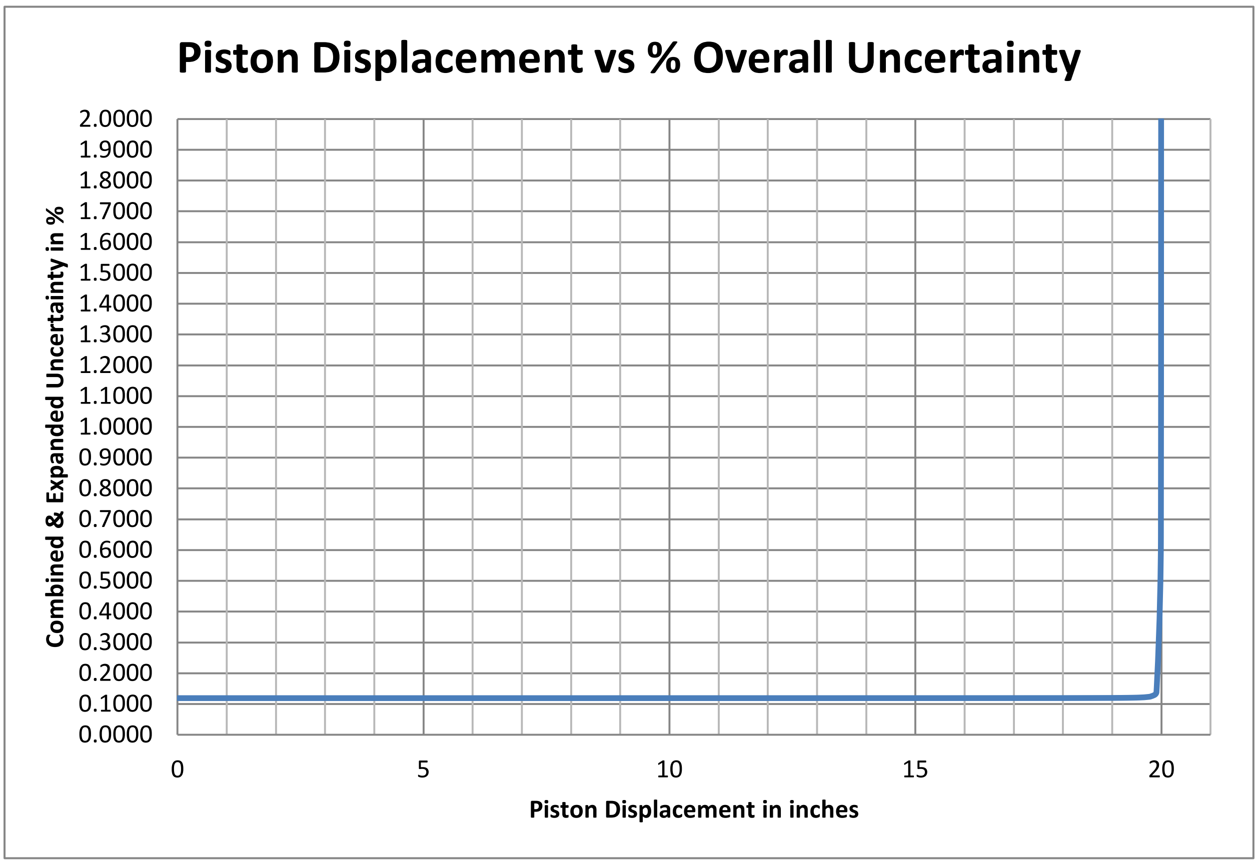


Figure 3.As the piston moves down the Cylinder V_{STD} decreases.



Graph 2. Effect of Piston Displacement on the Overall Uncertainy

From Graph 2 above it can be seen that as the piston travels down the cylinder the overall uncerterainty of the prover does not change significantly until at a piston displacement of 19.99 inches the uncertainty due to V_{STD} becomes significant and approaches ∞ as the piston displacement reaches 20 inches. This 50 gpm liquid flow calibrator has two flags that enable and disable the encoder and are spaced approximately 19.5 inches apart. There is approximately still 0.5 inches of travel left after the last flag is reached. Therefore, the constantly changing V_{STD} will not significantly influence the overall uncertainty of the stand.

The APSL estimated the volume of the uncertainty of the fluid in the piston cylinder element to within 12%. The uncertainty summary follows:

Uncertainty Summary

Type of Uncertainty: Relative Type B

Sensitivity Coefficient:

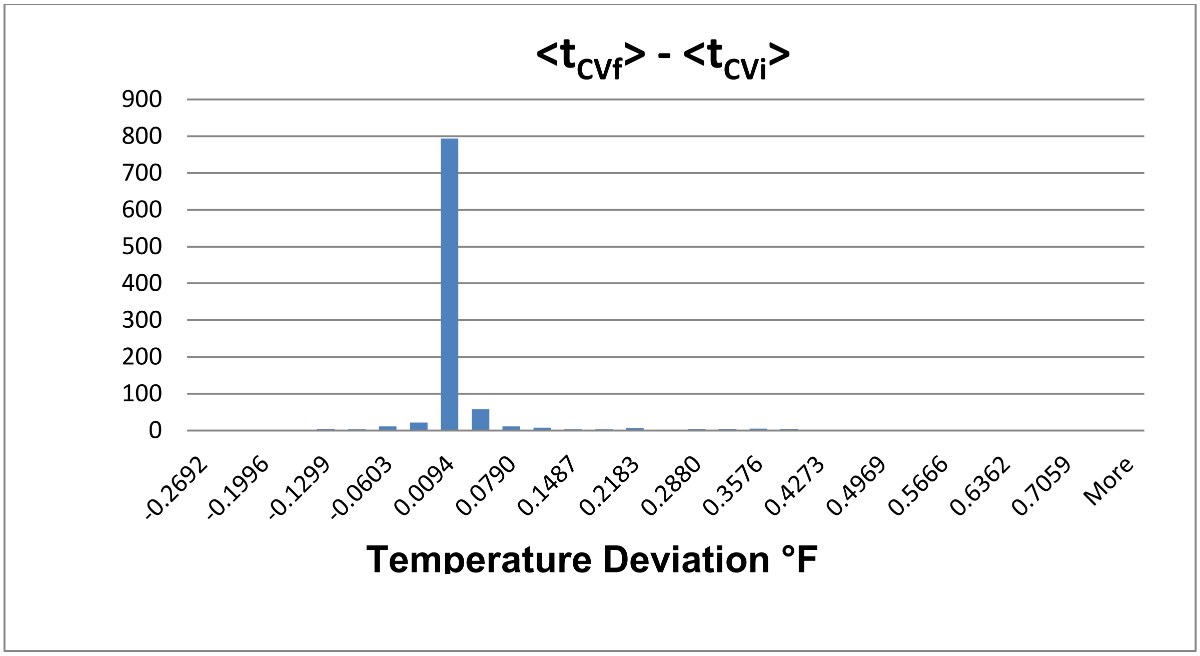
$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial V_{STD}} = \frac{V_{CV}}{V_{STD}} \cdot \frac{\beta \cdot \left(\left\langle t_{CVf} \right\rangle - \left\langle t_{CVi} \right\rangle\right)}{V_{STD} \cdot \left[1 + \beta \cdot \left(\overline{\overline{T}}_{MUT} - \overline{\overline{T}}_{STD}\right)\right] + V_{CV} \cdot \beta \cdot \left(\left\langle t_{CVf} \right\rangle - \left\langle t_{CVi} \right\rangle\right)}$$

Distribution: Rectangular

One Standard Uncertainty: $\frac{(20 \cdot in) \cdot \pi \cdot (6.00021573487587/2)^2 \cdot (12\%/100\%)}{2} = 33.9216 \cdot in^3$

Temperature Deviation between Connecting Volume Temperature at the End of Volume Delivery and Connecting Volume Temperature at the beginning of Volume Delivery, $\Delta\langle t \rangle = \left\langle t_{CVf} \right\rangle - \left\langle t_{CVi} \right\rangle$ [°F]

This 50 gpm stand has one temperature probe located in the connecting volume 7 inches from the delivery port of the piston cylinder and 18 inches from the MUT. The measurements from 38 FT0-4NIYS-1 turbine meters were selected because the model FT0-4NIYS-1 requires the longest measurement intervals of all meters calibrated on this 50 gpm flow stand. From these 38 meters 950 points were selected and the temperature at the start of the flow measurement $\left\langle t_{CVi} \right\rangle$ was recorded and the temperature measurement at the end of the flow measurement was $\left\langle t_{CVf} \right\rangle$ was recorded. This resulted in 950 temperature deviations from 950 flow measurements. The average temperature deviation out of these 950 measurements was 0.01°F. The distribution of these deviations is shown below in Graph 2.



Graph 2. Distribution of temperature deviations at the start of a flow measurement and at the end of the flow measurement on the 50 gpm piston prover

Connecting Volume Fluid Temperature at the End of Volume Delivery, $\langle t_{CVf} \rangle$ [°F]

The Marlin PRT installed in this 50 gpm stand has a tolerance of 0.1% as provided by the online product catalog [9]. The uncertainty for the calibrator fluid temperature measurement is given below in the uncertainty summary.

Uncertainty Summary:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial \langle t_{CVf} \rangle} = \frac{-V_{CV} \cdot \beta}{V_{STD}\left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right)\right] + V_{CV} \cdot \beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)}$

Distribution: Rectangular

One Standard Uncertainty: $\frac{68°F \cdot \left(0.1\% / 100\right)}{\sqrt{3}} = 0.039259818°\text{F}$

Connecting Volume Fluid Temperature at the Beginning of Volume Delivery, $\langle t_{CVi} \rangle$ [°F]

Similarly, uncertainty for the calibrator fluid temperature measurement is given below in the uncertainty summary.

Uncertainty Summary:

Type of Uncertainty: Relative Type B

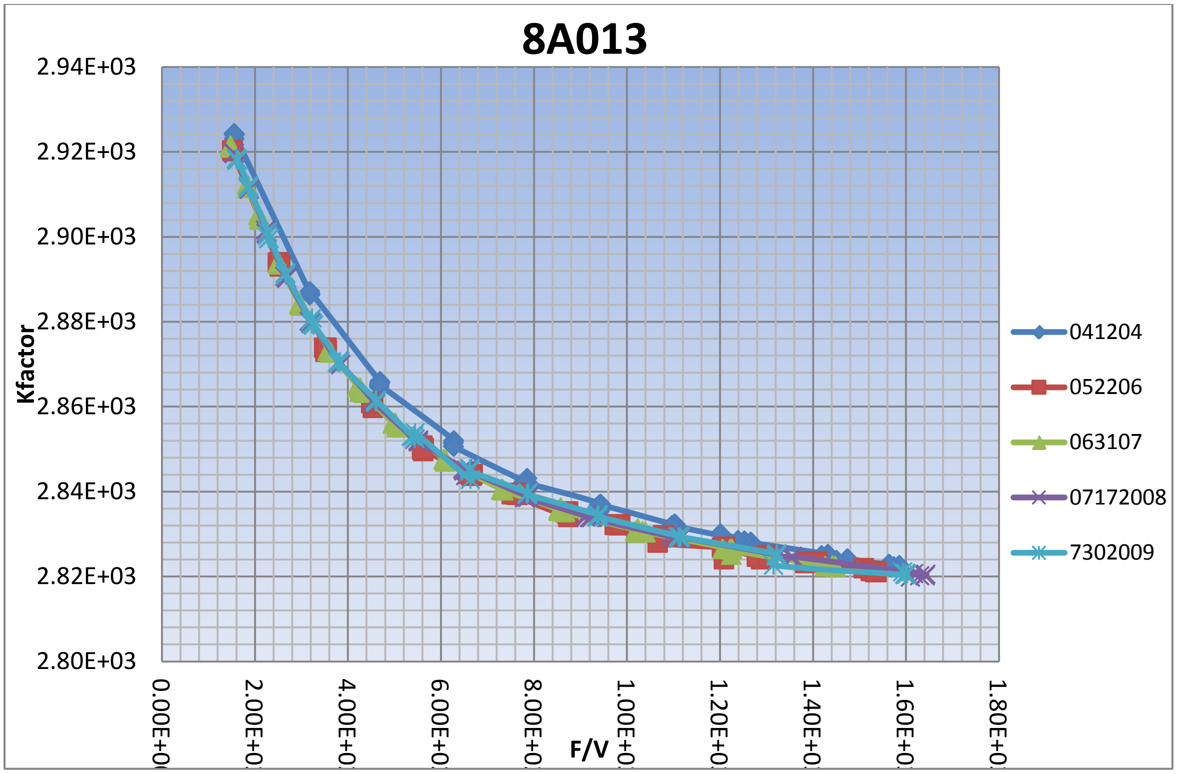
Sensitivity Coefficient: $$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial \langle t_{CVi} \rangle} = \frac{V_{CV} \cdot \beta}{V_{STD}\left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right)\right] + V_{CV} \cdot \beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)}$$

Distribution: Rectangular

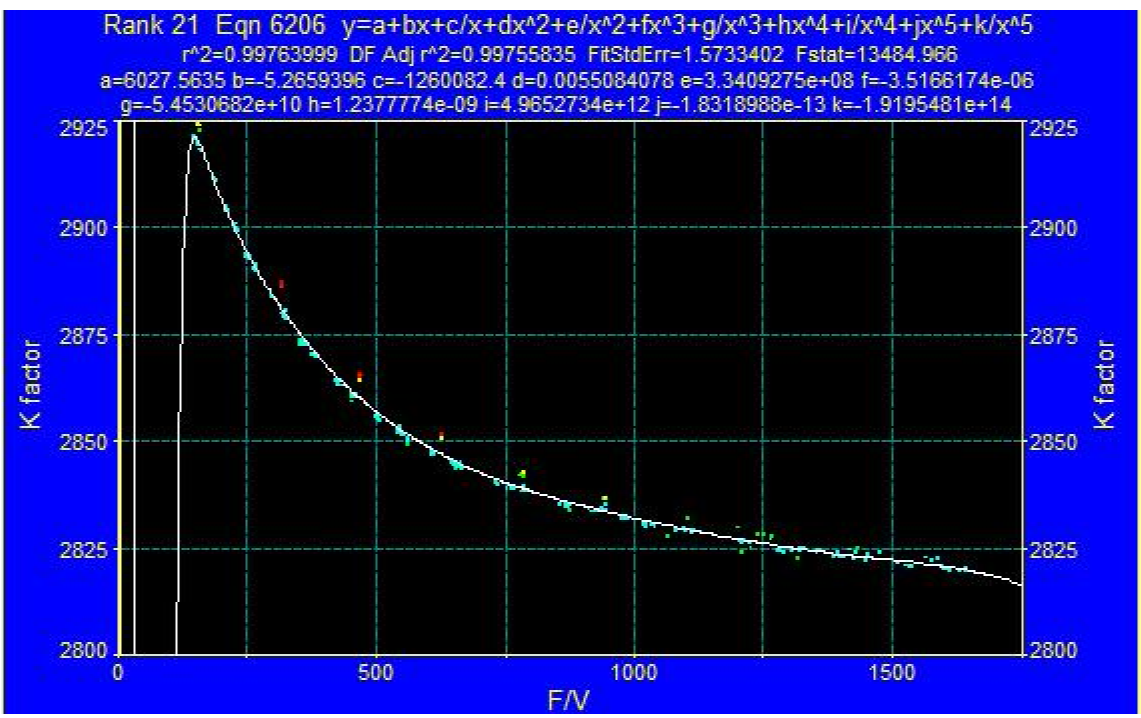
One Standard Uncertainty: $$\frac{68°F \cdot \left(0.1\% / 100\right)}{\sqrt{3}} = 0.039259818°\text{F}$$

Type A Turbine Meter K-Factor Uncertainty Due to Meter Repeatability and Reproducibility

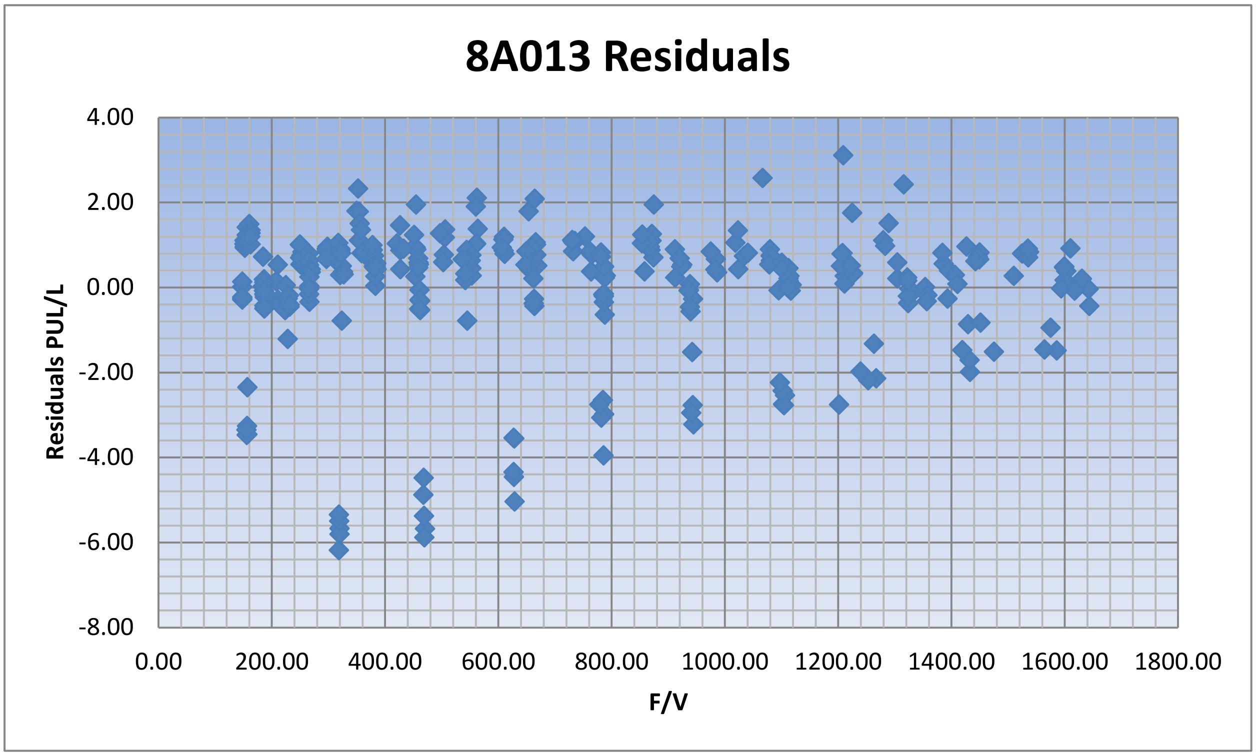
The following repeatability and reproducibility data for turbine meter 8A013 was taken over a period of 5 years for calibrations performed on April 12, 2004, May 22, 2006, June 13, 2007, July 17 2008, and July 30, 2009. The data was collected from data archived on v:\psl\liquidflow\MT50 SN_MT96090156\data in the following files: 8A013_041204.sav, 8A013_052206.sav, 8A013_061307.sav, 8A013_07172008.sav and 8A013_07302009.sav. This data was all collected into Excel, and TableCurve 2D ver. 5.0 was utilized to establish a trend line for the data taken over this six year period. The residuals from the trend line were then used to determine the repeatability/reproducibility of the meter. Below is a graph of the data from the six years of collected data for turbine flow meter 8A013.



Graph 3. Calibrations of Turbine Flow Meter 8A013 over 5 years



Graph 4. Trend line for Turbine Flow Meter SN: 8A013



Graph 5. Scatter Plot of Turbine Flow Meter SN: 8A013 Residuals of 5 calibrations performed over 5 years

Using the Equation shown derived from TableCurve 2D 5.0:

Eqn# 6206 y=a+bx+c/x+dx^2+e/x^2+fx^3+g/x^3+hx^4+i/x^4+jx^5+k/x^5

where:

a= 6027.563487368546
b= -5.265939574767581
c= -1260082.425247799
d= 0.005508407797475202
e= 334092753.0823971
f= -3.516617356591668E-06
g= -54530681672.39389
h= 1.237777392406586E-09
i= 4965273406801.692
j= -1.831898821453802E-13
k= -191954806562639.9

and taking the % deviation of the residuals of 390 points from the trend line established by Eqn# 6206 the %Repeatability/%Reproducibility is obtained at one standards uncertainty to be 0.054941%.

Uncertainty Summary

Type of Uncertainty: Relative Type A

Sensitivity Coefficient: K_{MUT}^{-1}

Distribution: Normal

One Standard Uncertainty: $\sqrt{\frac{(\mathrm{x}_{t1}-x_1)^2+(x_{t2}-x_2)^2+\cdot\cdot(x_{tn}-x_n)^2}{n-1}}=0.05429275\%$

Uncertainty in Meter K-Factor, K_{MUT} Due to Kinematic Viscosity

The uncertainty contribution that kinematic viscosity can have on meter K-Factor, K_{MUT} may be significant or it may not be significant. It depends on the flow rate, meter size and the viscosity of the liquid flowing through the meter.

Typically, the APSL Liquid Flow Laboratory will fit the data for Meter K-Factor, K_{MUT} versus Frequency/Viscosity (F/V). In the repeatability uncertainty section above TableCurve 2D v5.0 was used to fit the data for a turbine meter SN: 8A013 using TableCurve Equation 6206. Using this equation Graph 6 illustrates how a change in F/V relates to a change in Meter K-Factor, K_{MUT}

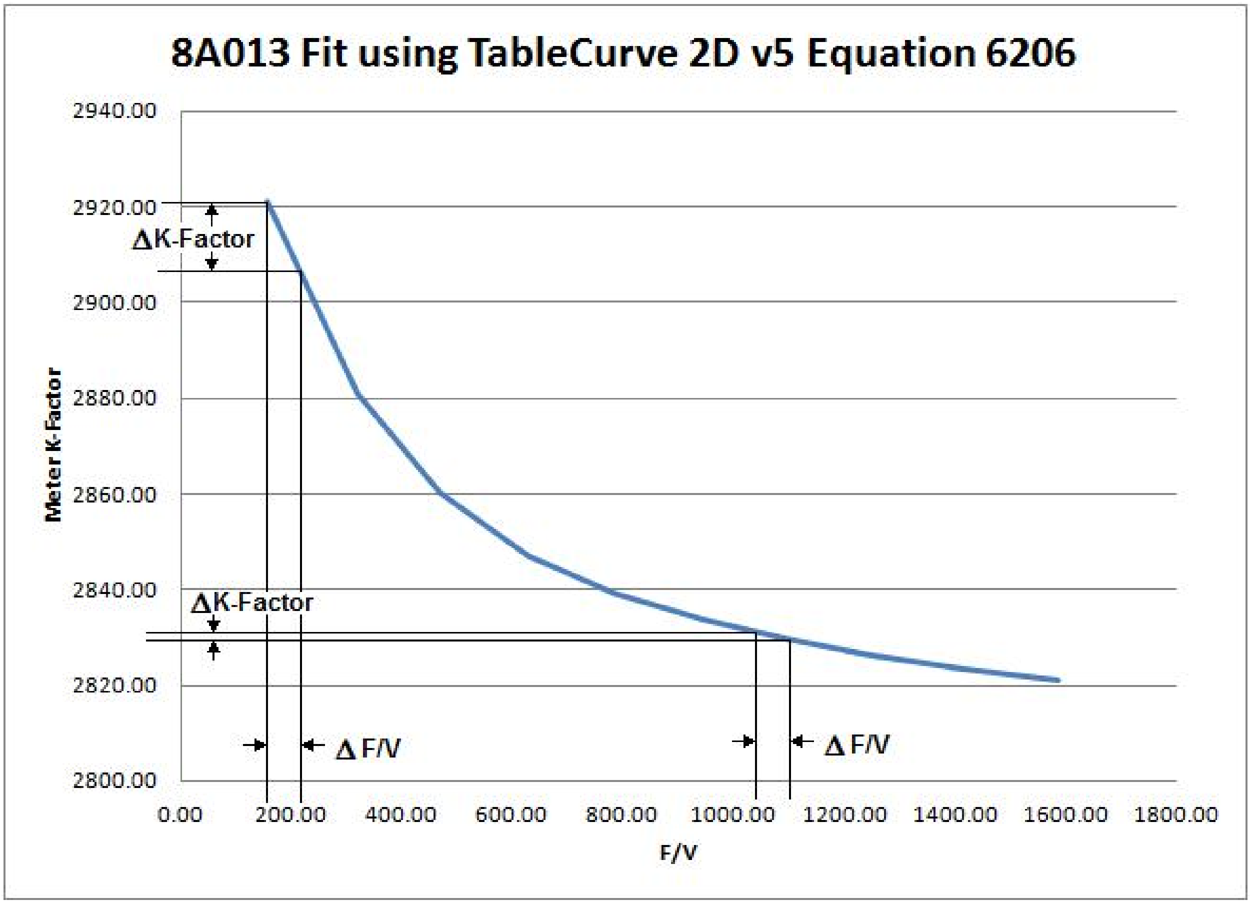
Equation 6206: $y=a+b\cdot x+\frac{c}{x}+d\cdot x^2+\frac{e}{x^2}+f\cdot x^3+\frac{g}{x^3}+h\cdot x^4+\frac{i}{x^4}+j\cdot x^5+\frac{k}{x^5}$

Where

$y=K_{MUT}=f(x),\ x=\frac{F}{V}$

and

a= 6027.563487368546
b= -5.265939574767581
c= -1260082.425247799
d= 0.005508407797475202
e= 334092753.0823971
f= -3.516617356591668E-06
g= -54530681672.39389
h= 1.237777392406586E-09
i= 4965273406801.692
j= -1.831898821453802E-13
k= -191954806562639.9



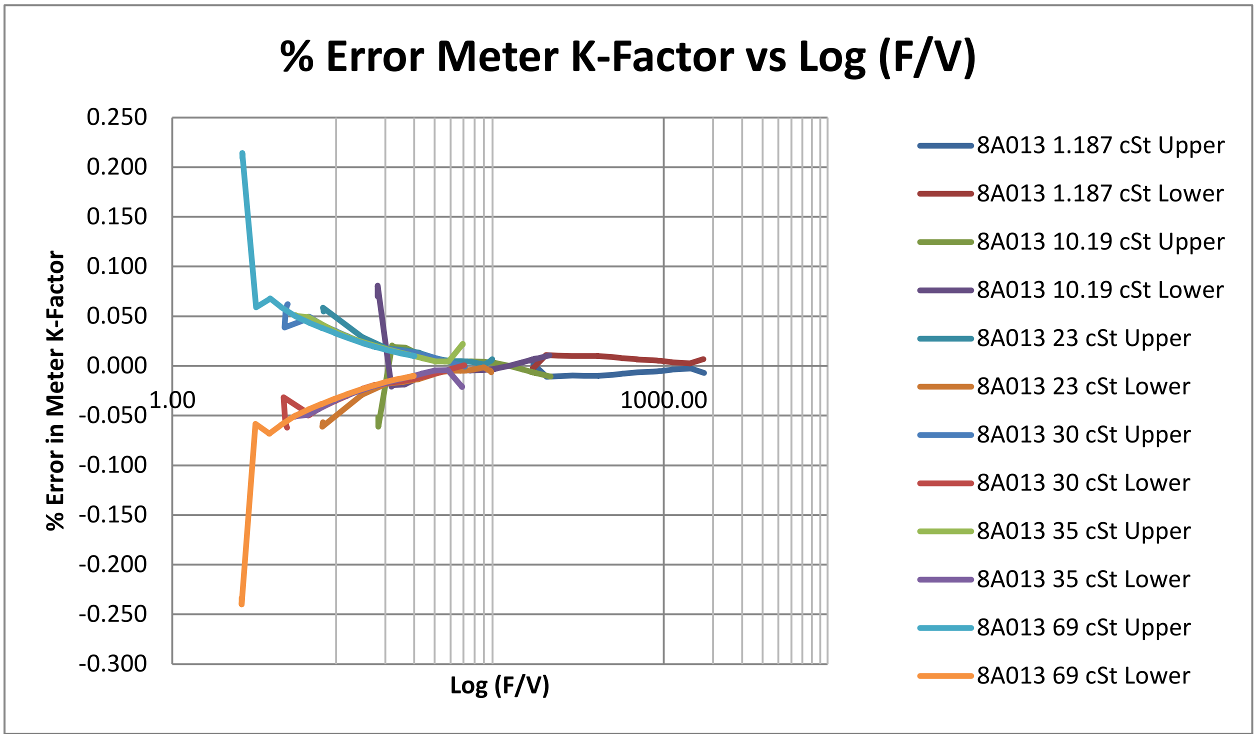
Graph 6 demonstrates how a change in (F/V) influences and error in Meter K-Factor.

It is obvious from Graph 6 that the same change in F/V will cause a larger change in Meter K-Factor wherever the slope of the fit is the largest.

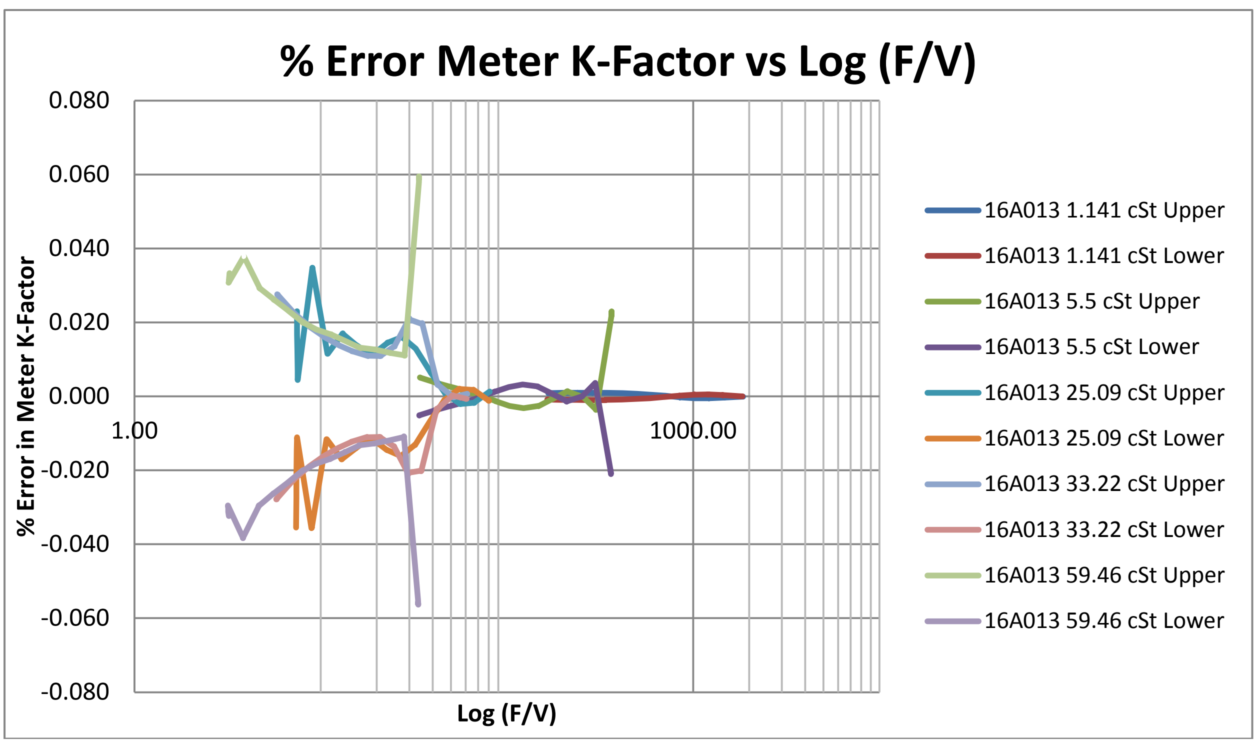
The software on this 50 gpm stand uses a temperature vs. viscosity table that is derived using a bath and viscometer, and fits the viscosity data using Andrade's equation. The software provider estimates the accuracy of this viscosity determination to be ±1%. Assuming a normal distribution one standard uncertainty is 0.5%.

A ±0.5% error in viscosity, V will directly result in a ±0.5% error in F/V. Graphs 7 and 8 were generated by varying the actual F/V taken from actual meter data by ±0.5% for an FT8 and an FT16 Turbine Meter over different viscosities.

The ±0.5% F/V numbers were then run back through the equation fit generated by TableCurve 2D v5.0 to determine the Meter K-Factor for both a +F/V error and a –F/V error. The % error in Meter K-Factor was then plotted against Log(F/V). The Log plot was used to stretch out the scale on the F/V axis, making the change in meter K-factor more visible.



Graph 7. %Error in Meter K-Factor vs Log(F/V) for Meter SN: 8A013 at 1.187 cSt, 10.19 cSt, 23 cSt, 30 cSt, 35 cSt, and 69 cSt.



Graph 8. %Error in Meter K-Factor vs Log(F/V) for Meter SN: 16A013 at 1.14 cSt, 5.5 cSt, 25.09 cSt, 33.22 cSt, and 59.46 cSt.

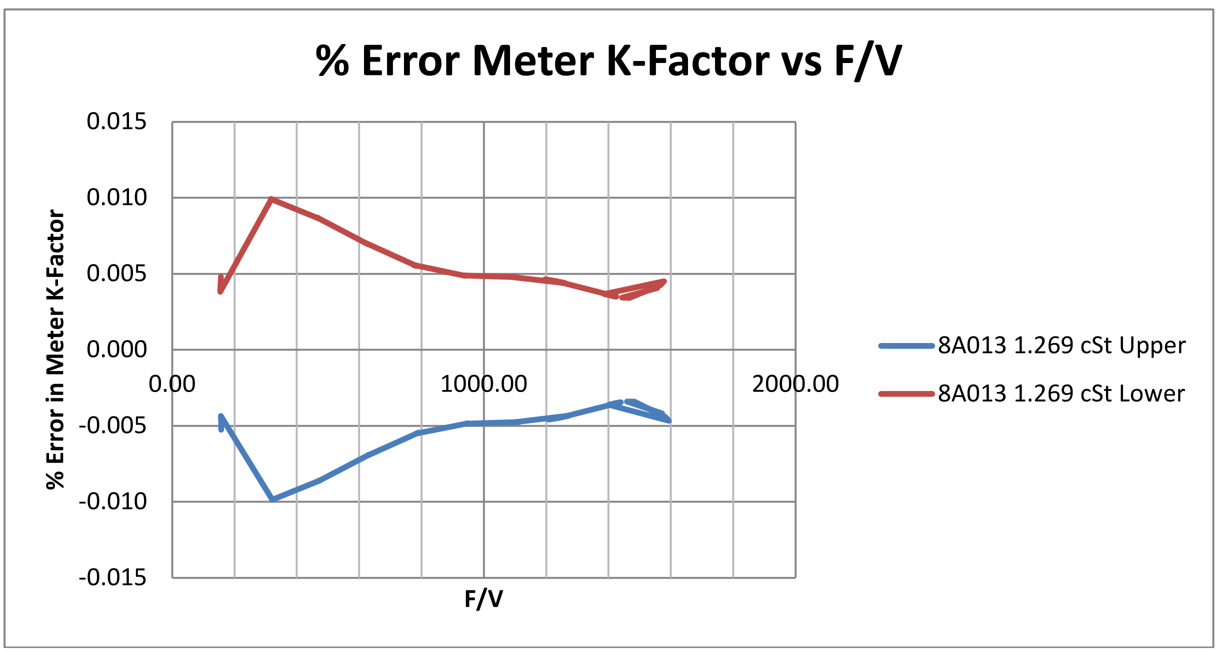
Graphs 7 and 8 illustrate how meter size and viscosity influence the %Error in Meter K-Factor. Typically the steepest slope of the curve fit is at the lowest F/V; however, in Graph 8 for the 16A013 at 59.46 cSt the largest %Error in Meter K-Factor occurs at the highest F/V for the 59.46 cSt run. This larger error occurs at the end of the of the 59.26 cSt run because the +F/V value used to determine Meter K-factor lies outside the original data set that was used to generate the curve fit using TableCurve 2D v5.0. This error also can occur for –F/V. Using the largest slope in the equation curve fit will find the largest % error in meter K-factor only when there are no F/V values used that lie outside the original data set used to generate the curve fit.

There are five methods that can be employed when determining the uncertainty in K-factor caused by errors in viscosity: 1) ignore the error in due to viscosity; 2) cut off the steep sloped portion of the curve fit that causes significant errors in the overall uncertainty budget; 3) find where the worst case % error due to viscosity is and include this in the overall budget; 4) take an average % error in viscosity and include this in the overall budget and 5) provide an overall point-by-point uncertainty budget that includes the % error due to viscosity.

Method 3 is the most conservative, Method 4 is the most practical and Method 5 is the most revealing.

Since this run for meter SN 8A013 used for this uncertainty analysis was performed in a hydrocarbon media at 1.269 cSt the worst case contribution is selected because it is the most conservative method and will yield an insignificant % error in meter K-factor due to the low viscosity liquid.

Graph 9 is a plot of % Error Meter K-Factor vs F/V for Meter SN: 8A013 in 1.269 cSt.



Graph 9. Plot of % Meter K-Factory vs F/V for Meter SN: 8A0131 at 1.269 cSt.

From the data set for the 8A103 Turbine Meter in 1.269 cSt the worst case maximum % error is 0.00992% and therfore 0.00992% will be used for the worst case scenario.

Uncertainty Summary for effect of Viscosity on Meter K-Factor (Meter Specific).

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: K_{MUT}^{-1}

Distribution: Normal

One Standard Uncertainty: $\left(\frac{2881.2364 - 2880.6665}{2880.6665}\right) \times 100\% = 0.00992\%$

Combined and Expanded Uncertainty

Combined Uncertainty

The combined uncertainty is obtained by root sum squaring all the uncertainty contributions as shown below. Note that the temperature terms are assumed all to be correlated and their uncertainties were simply summed together to obtain a conservative worst case uncertainty.

$$U_C = \sqrt{u_{p_c}^2 + u_{t_C}^2 + u_{P_{MUT}}^2 + u_{t_{MUT}}^2 + u_{K_C}^2 + u_{\alpha_{ENC}}^2 + \left(u_{T_{AMB}} + u_{T_{STD}} + u_{T_{MUT}} + u_{\langle t_{CVf}\rangle} + u_{\langle t_{CVi}\rangle}\right)^2 + u_{\alpha_T}^2 + u_{\alpha_P}^2 + u_{P_{STD}}^2 + u_{\beta}^2 + u_{V_{CV}}^2 + u_{V_{STD}}^2 + u_{\eta}^2 + u_R^2}$$

$$U_C = 0.0596219\%$$

Expanded Uncertainty

The expanded uncertainty is achieved by multiplying the above calculation by a coverage factor k=2 to achieve a 95% level of confidence.

$$U_E = kU_C = 2(0.0596219\%) = 0.1192439\%$$

Table 6 is the spreadsheet used for the calculation of the uncertainty budget for this 50 gpm machine using turbine meter SN: 8A013 in 1.269 cSt liquid media.

				Distribution							
Influence Quantity	Variable	Value	Units	Normal	Rectangular	One Standard Uncertainty	$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial x}$ Sensitivity Coefficient	Sensitivity Value	Std Unc. (1σ)	Variance σ^2	Std Unc. 2σ %
Calibrator Encoder Pulses	P_C	2740.81	[pulses]	0.251359676		0.125679838	$-P_C^{-1}$	-3.65E-04	4.59E-05	2.10E-09	0.00917
Calibrator Time	t_C	1	s		0.00005	2.88675E-05	t_C^{-1}	1	2.89E-05	8.33E-10	0.00577
Flow Meter Pulses	P_{MUT}	2740.81	[pulses]		0.000	0	P_{MUT}^{-1}	3.65E-04	0.00E+00	0.00E+00	0.00000
Flow Meter Time	t_{MUT}	1	s		0.00005	2.88675E-05	$-t_{MUT}^{-1}$	-1	2.89E-05	8.33E-10	0.00577
Water Draw Encoder Pulses (Water Draw)	K_C	**2740.81**	**[pulses]**	**1.140176918**		**0.57008846**	K_C^{-1}	**3.65E-04**	2.08E-04	**4.33E-08**	**0.04160**
Encoder Linear Thermal Expansion Coefficient	α_{ENC}	4.44E-06	°F^{-1}		5.56E-07	3.2075E-07	$\frac{-(T_{AMB}-T_{REF})}{[1-\alpha_{ENC}\cdot(T_{AMB}+T_{REF})]}$	0.00	0.00E+00	0.00E+00	0.00000
Ambient Temperature Measurement	T_{AMB}	68.00	°F		3.600	2.078460969	$\frac{-\alpha_{ENC}}{[1-\alpha_{ENC}\cdot(T_{AMB}+T_{REF})]}$	-4.44E-06	9.23E-06	8.53E-11	0.00185
Tube Area Thermal Expansion Coefficient	α_T	1.766E-05	°F^{-1}		6.244E-07	3.60483E-07	$\frac{-(T_{STD}-T_{REF})}{[1-\alpha_T\cdot(T_{STD}-T_{REF})]}$	-5.500E-01	1.98E-07	3.93E-14	0.00004
Average Temperature of the Fluid in Standard	$\overline{T}_{STD}$	68.55	°F		0.069	0.039577361	$-\frac{\alpha_T\cdot[V_{STD}+V_{STD}\cdot\beta\cdot(\overline{T}_{MUT}-T_{REF})+V_{CV}\cdot\beta\cdot(\langle t_{CVf}\rangle-\langle t_{CVi}\rangle)]-V_{STD}\cdot\beta}{[1-\alpha_T\cdot(\overline{T}_{STD}+T_{REF})]\cdot[V_{STD}+V_{STD}\cdot\beta\cdot(\overline{T}_{MUT}-T_{STD})+V_{CV}\cdot\beta\cdot(\langle t_{CVf}\rangle-\langle t_{CVi}\rangle)]}$	5.276E-04	2.09E-05	4.36E-10	0.00418
Reference Temperature	T_{REF}	**68.00**	**°F**	**0**	**0**	0	$\frac{\alpha_{ENC}\cdot[1-\alpha_T\cdot(\overline{T}_{STD}+T_{REF})]+\alpha_T\cdot[1-\alpha_{ENC}\cdot(T_{AMB}-T_{REF})]}{[1-\alpha_T\cdot(\overline{T}_{STD}+T_{REF})]\cdot[1-\alpha_{ENC}\cdot(T_{AMB}+T_{REF})]}$	**2.22E-05**	0.00E+00	0.00E+00	0.00000
Volume Pressure Expansion Coefficient of Flow Tube	α_P	1.14286E-06	in^2/lbf		6.63E-12	3.82899E-12	$-\frac{(P_{STD}-P_{REF})}{[1+\alpha_P\cdot(P_{STD}-P_{REF})]}$	-60.004	2.30E-10	5.28E-20	0.00000
Fluid Pressure Measurement	P_{STD}	60.00	lbf/in^2		3.2	1.847520861	$\frac{-\alpha_P}{[1-\alpha_P\cdot(P_{STD}+P_{REF})]}$	-1.143E-06	2.11E-06	4.46E-12	4.22E-04
Standard Pressure (gauge)	P_{REF}	**0.00**	**lbf/in^2**	**0**	**0**	0	$\frac{\alpha_P}{[1-\alpha_P\cdot(P_{STD}+P_{REF})]}$	**-1.143E-06**	0.00E+00	0.00E+00	0.00000
Thermal Expansion of the Fluid Media	β	5.4420E-04	°F^{-1}	8.60E-05		4.29921E-05	$-\frac{V_{STD}\cdot(\overline{T}_{MUT}-\overline{T}_{STD})+V_{CV}\cdot(\langle t_{CVf}\rangle-\langle t_{CVi}\rangle)}{V_{STD}\cdot[1+\beta\cdot(\overline{T}_{MUT}-\overline{T}_{STD})]+V_{CV}\cdot\beta\cdot(\langle t_{CVf}\rangle-\langle t_{CVi}\rangle)}$	5.4947E-01	2.36E-05	5.58E-10	0.00472
Averge Temperature of Fluid through the MUT	$\overline{T}_{MUT}$	69.10			0.069	0.039894904	$\frac{-\beta\cdot V_{STD}}{V_{STD}\cdot[1+\beta\cdot(\overline{T}_{MUT}-\overline{T}_{STD})]+V_{CV}\cdot\beta\cdot(\langle t_{CVf}\rangle-\langle t_{CVi}\rangle)}$	-5.4404E-04	2.17E-05	4.71E-10	0.00434
Connecting Volume	V_{CV}	20.522	in^3	2.46		1.231320722	$\frac{\beta\cdot(\langle t_{CVf}\rangle-\langle t_{CVi}\rangle)}{-[V_{STD}\cdot[1+\beta\cdot(\overline{T}_{MUT}-\overline{T}_{STD})]+V_{CV}\cdot\beta\cdot(\langle t_{CVf}\rangle-\langle t_{CVi}\rangle)]}$	9.6201E-09	1.18E-08	1.40E-16	0.00000
Standard Volume	V_{STD}	565.53	**in^3**	67.86		33.93164061	$\frac{V_{CV}}{V_{STD}}\cdot\frac{\beta\cdot(\langle t_{CVf}\rangle-\langle t_{CVi}\rangle)}{V_{STD}\cdot[1+\beta\cdot(\overline{T}_{MUT}-\overline{T}_{STD})]+V_{CV}\cdot\beta\cdot(\langle t_{CVf}\rangle-\langle t_{CVi}\rangle)}$	-3.491E-10	1.18E-08	1.40E-16	0.00000
Temperature of CV at End of Volume Delivery	$\langle t_{CVf}\rangle$	68.00	°F		0.068	0.039259818	$\frac{-V_{CV}\cdot\beta}{V_{STD}[1+\beta\cdot(\overline{T}_{MUT}-\overline{T}_{STD})]+V_{CV}\cdot\beta\cdot(\langle t_{CVf}\rangle-\langle t_{CVi}\rangle)}$	-1.974E-05	7.75E-07	6.01E-13	0.00016
Temperature of CV at Beginning of Volume Delivery	$\langle t_{CVi}\rangle$	68.01	°F		0.068	0.039265592	$\frac{V_{CV}\cdot\beta}{V_{STD}[1+\beta\cdot(\overline{T}_{MUT}-\overline{T}_{STD})]+V_{CV}\cdot\beta\cdot(\langle t_{CVf}\rangle-\langle t_{CVi}\rangle)}$	1.9742E-05	7.75E-07	6.01E-13	0.00016
Error in Meter K-Factor due to Kinematic Viscosity	K_{MUT}	2912.372	pulses/L	0.57778714		0.28889357	K_{MUT}^{-1}	0.000343363	9.92E-05	9.84E-09	0.01984
Meter Repeatability & Reproduceabilty (Type A) 0.1-1 gpm	K_{MUT}	2912.372	pulses/L	3.162413738		1.581206869	K_{MUT}^{-1}	0.000343363	5.43E-04	2.95E-07	0.10859
									Combined Variance:	3.56E-07	
									Combined Uncertainty (k=1):	0.0005963	
Displacement of Piston	0.000								**Expanded Uncertainty (k=2):**	**0.1193 %**	

Table 6. Uncertainty Budget for APSL 50 gpm Liquid Flow Calibrator using Turbine Meter SN: 8A013 in 1.269 cSt hydrocarbon.

Conclusions

Significance of Connecting Volume

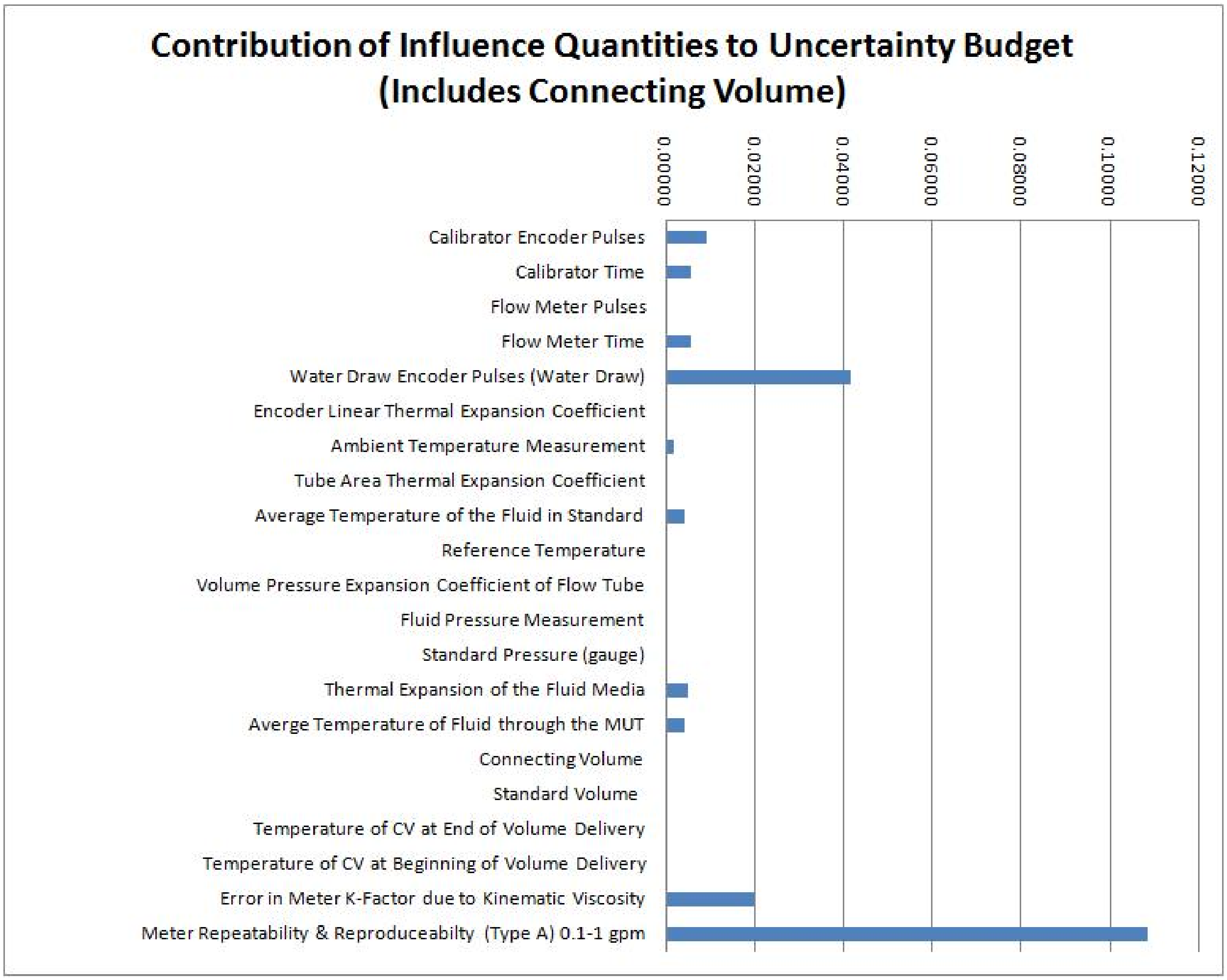
Table 7 below is an uncertainty analysis performed by the same APSL 50 gpm prover without considering connecting volume.

				Distribution							
Influence Quantity	Variable	Value	Units	Normal	Rectangular	One Standard Uncertainty	$\frac{1}{K_{MUT}}\cdot\frac{\partial K_{MUT}}{\partial x}$ Sensitivity Coefficient	Sensitivity Value	Std Unc. (1σ)	Variance σ^2	Std Unc. 2σ %
Calibrator Encoder Pulses	P_C	2740.81	[pulses]	0.251359676		0.125679838	$-P_C^{-1}$	-3.65E-04	-4.59E-05	2.10E-09	0.00917
Calibrator Time	t_C	1	s		0.00005	2.88675E-05	t_C^{-1}	1	2.89E-05	8.33E-10	0.00577
Flow Meter Pulses	P_{MUT}	2740.81	[pulses]		0.000	0	P_{MUT}^{-1}	3.65E-04	0.00E+00	0.00E+00	0.00000
Flow Meter Time	t_{MUT}	1	s		0.00005	2.88675E-05	$-t_{MUT}^{-1}$	-1	-2.89E-05	8.33E-10	0.00577
Water Draw Encoder Pulses (Water Draw)	K_C	**2740.81**	**[pulses]**	**1.140176918**		**0.57008846**	K_C^{-1}	**3.65E-04**	**2.08E-04**	**4.33E-08**	**0.04160**
Encoder Linear Thermal Expansion Coefficient	α_{ENC}	4.44E-06	°F⁻¹		5.56E-07	3.2075E-07	$\frac{-(T_{AMB}-T_{REF})}{[1-\alpha_{ENC}\cdot(T_{AMB}+T_{REF})]}$	0.00E+00	0.00E+00	0.00E+00	0.00000
Ambient Temperature Measurement	T_{AMB}	68.00	°F		3.600	2.078460969	$\frac{-\alpha_{ENC}}{[1-\alpha_{ENC}\cdot(T_{AMB}+T_{REF})]}$	-4.44E-06	-9.23E-06	8.53E-11	0.00185
Tube Area Thermal Expansion Coefficient	α_T	1.766E-05	°F⁻¹		6.244E-07	3.60483E-07	$\frac{-(T_{STD}-T_{REF})}{[1-\alpha_T\cdot(T_{STD}-T_{REF})]}$	-0.55	-1.98E-07	3.93E-14	0.00004
Fluid Temperature Measurement (Master = Meter)	$\bar{T}_{STD}$	68.55	°F		0.069	0.039577361	$\frac{\alpha_T}{(1-\alpha_T\cdot(T_{STD}-T_{REF}))}$	1.77E-05	6.99E-07	4.89E-13	0.00014
Standard Temperature	T_{REF}	**68.00**	°F	-	-	-	-	-	-	-	-
Volume Pressure Expansion Coefficient of Flow Tube	α_P	1.14286E-06	in²/lbf		6.63E-12	3.82899E-12	$-\frac{(P_{STD}-P_{REF})}{[1+\alpha_P\cdot(P_{STD}-P_{REF})]}$	-60.00	-2.30E-10	5.28E-20	0.00000
Fluid Pressure Measurement	P_{STD}	60.00	lbf/in²		3.2	1.847520861	$\frac{-\alpha_P}{[1-\alpha_P\cdot(P_{STD}+P_{REF})]}$	-1.14E-06	-2.11E-06	4.46E-12	4.22E-04
Standard Pressure (gauge)	P_{REF}	**0.00**	**lbf/in²**	-	-	-	-	-	-	-	-
Error in Meter K-Factor due to Kinematic Viscosity	K_{MUT}	**2912.37**	**pulses/L**	**0.57778714**		0.28889357	K_{MUT}^{-1}	**0.00034336**	9.92E-05	9.84E-09	0.01984
Meter Repeatability & Reproduceabilty (Type A) 1.0-10 gpm	K_{MUT}	**2912.37**	[pulses]	**3.162413738**		1.581206869	K_{MUT}^{-1}	**0.00034336**	5.43E-04	2.95E-07	0.10859
							Combined Variance:			3.52E-07	
							Combined Uncertainty (k=1):			0.0005931	
							Expanded Uncertainty (k=2):			**0.1186 %**	

Table 7. Uncertainty on 50 gpm Liquid Flow Calibrator without considering Connecting Volume.

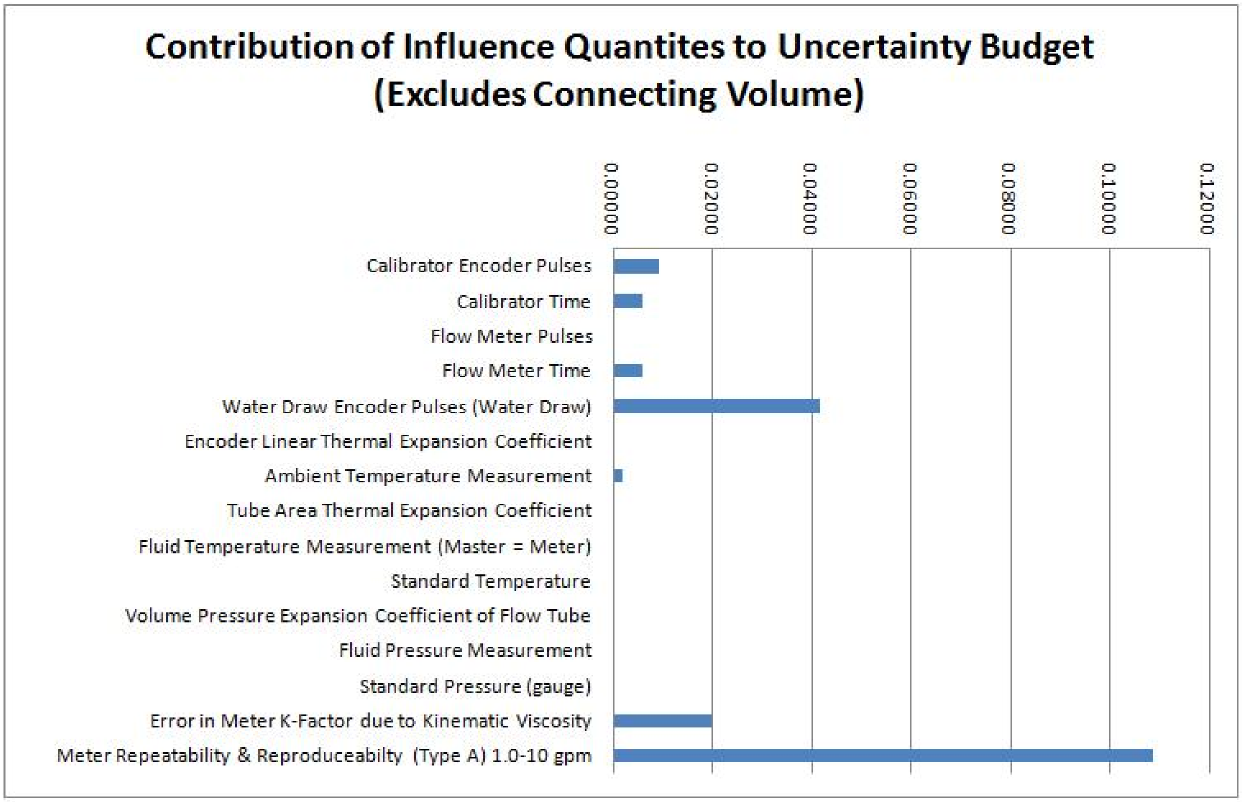
By comparing the uncertainty budget including connecting volume (Table 7) to the uncertainty budget not including connecting volume (Table 8) the overall combined and expanded (k=2) uncertainty is 0.1192% to 0.1186% respectively. From the analysis for the APSL 50 gpm liquid flow calibrator used in this uncertainty analysis the assumption that the density of the liquid media in the stand is the same at the delivery port of the piston cylinder and the meter under test is a good assumption and the contribution of connecting volume is insignificant.

Graph 9 shows the overall uncertainty contribution of each influence quantity when connecting volume is included in the uncertainty budget.



Graph 9 Influence Quantities and their contribution to error in overall Meter K-Factor for Uncertainty Budget Including Connecting Volume.

Graph 10 shows the overall uncertainty contribution of each influence quantity when connecting volume is excluded in the Uncertainty Budget.



Graph 10. Influence Quantities and their contribution to error in overall Meter K-Factor for Uncertainty Budget Excluding Connecting Volume.

The overall significance of connecting volume in an uncertainty budget depends on the design of the particular liquid flow piston prover under consideration. Provers with large volumes between the discharge port of the piston cylinder element and the meter under test will have larger contributions to the uncertainty budget. Also temperature control plays a large factor as well.

Viscosity

As shown in this paper, the influence of viscosity at the meter and its contribution to the overall uncertainty can be significant especially for high viscosity fluids flowing through small meters at low flow rates. The liquid flow community does not have a standard method for addressing the influence of viscosity in uncertainty budgets. The method chosen may depend on whether the laboratory chooses to be ignorant, safe, conservative, practical, or concise.

Appendices

A. Connecting Volume Derivation:

$$\overline{\dot{M}}_{MUT} = \overline{\dot{M}}_{STD} \text{ (Continuity Equation)} \quad (1)$$

Where:

$\overline{\dot{M}}_{MUT}$ = Average mass flow rate through the Meter Under Test (MUT)
$\overline{\dot{M}}_{STD}$ = Average mass flow rate from the Standard Liquid Flow Prover

From the relationship $M = \frac{V}{\rho}$ (2)

Where

M = Mass
V = Volume
ρ = Density

The average Volumetric Flow Rate $\overline{\dot{V}}$ is given by

$$\frac{V}{t_C} = \frac{M}{\rho \cdot t_C} = \overline{\dot{V}} \quad (3)$$

Therefore,

$\overline{\dot{V}}_{STD} = \frac{M_{STD}}{\rho_{STD} \cdot t_C}$, which is the average volumetric flow rate from the Standard Prover (4)

$\overline{\dot{V}}_{MUT} = \frac{M_{MUT}}{\rho_{MUT} \cdot t_C}$, which is the average volumetric flow rate through the Meter Under Test (5)

From the ratio of equations 4 and 5 the following is obtained

$$\frac{\overline{\dot{V}}_{MUT}}{\overline{\dot{V}}_{STD}} = \left(\frac{M_{MUT}}{\rho_{MUT} \cdot t_C}\right) \Big/ \left(\frac{M_{STD}}{\rho_{STD} \cdot t_C}\right) = \frac{\rho_{STD}}{\rho_{MUT}} \quad (6)$$

And solving for $\overline{\dot{V}}_{MUT}$ the following relationship is obtained.

$$\overline{\dot{V}}_{MUT} = \overline{\dot{V}}_{STD} \cdot \left(\frac{\rho_{STD}}{\rho_{MUT}} \right) \tag{7}$$

This equation is derived assuming that the temperature is stable throughout the connecting volume, which may not actually be the case. In an effort to model the impact of temperature instabilities in the connecting volume J. Latsko and J. Winchester account for the connecting volume and temperature instabilities using the following relationship.

$$M_{STD} + M_{CVi} = M_{MUT} + M_{CVf} \text{ (Continuity Equation)} \tag{8}$$

M_{STD} = Mass of liquid in the Standard Prover
M_{CVi} = The mass of the liquid in the connecting volume at the initiation of volume delivery
M_{MUT} = Mass of liquid in the Meter Under Test
M_{CVf} = The mass of the liquid in the connecting volume at the end of volume delivery

From the fact that $M = \rho \cdot V$ (9)

Equation 8 becomes

$$\rho_{STD} \cdot V_{STD} + \rho_{CVi} \cdot V_{CV_i} = \rho_{MUT} \cdot V_{MUT} + \rho_{CVf} \cdot V_{CVf} \tag{10}$$

To obtain flow rate we divide by the delivery time t_C

$$\frac{\overline{\rho}_{STD} \cdot V_{STD}}{t_C} + \frac{\rho_{CVi} \cdot V_{CV}}{t_C} = \frac{\overline{\rho}_{MUT} \cdot V_{MUT}}{t_C} + \frac{\rho_{CVf} \cdot V_{CV}}{t_C} \tag{11}$$

$$\overline{\dot{V}}_{MUT} = \frac{V_{MUT}}{t_C} \text{ and } \overline{\dot{V}}_{STD} = \frac{V_{STD}}{t_C} \tag{12}$$

Therefore,

$$\frac{\overline{\rho}_{MUT} \cdot V_{MUT}}{t_C} = \frac{\overline{\rho}_{STD} \cdot V_{STD}}{t_C} + \frac{\rho_{CVi} \cdot V_{CV}}{t_C} - \frac{\rho_{CVf} \cdot V_{CV}}{t_C} \tag{13}$$

This reduces to

$$\overline{\dot{V}}_{MUT} = \frac{1}{\overline{\rho}_{MUT}} \cdot \left[\overline{\dot{V}}_{STD} \cdot \overline{\rho}_{STD} + V_{CV} \cdot \left(\frac{\rho_{CVi} - \rho_{CVf}}{t_C} \right) \right] \tag{14}$$

or

$$\overline{\dot{V}}_{MUT} = \overline{\dot{V}}_{STD} \cdot \frac{\overline{\rho}_{STD}}{\overline{\rho}_{MUT}} + \frac{V_{CV} \cdot (\rho_{CVi} - \rho_{CVf})}{\overline{\rho}_{MUT} \cdot t_C} \tag{15}$$

Factoring out $\overline{\dot{V}}_{STD}$ of the right hand side of equation 15 the following relationship is obtained

$$\overline{\dot{V}}_{MUT} = \overline{\dot{V}}_{STD} \cdot \left[\frac{\overline{\rho}_{STD}}{\overline{\rho}_{MUT}} + \frac{V_{CV} \cdot (\rho_{CVi} - \rho_{CVf})}{\overline{\dot{V}}_{STD} \cdot \overline{\rho}_{MUT} \cdot t_C} \right] \tag{16}$$

Given that $\overline{\dot{V}}_{STD} = \frac{V_{STD}}{t_C}$ then

$$\overline{\dot{V}}_{MUT} = \overline{\dot{V}}_{STD} \cdot \left[\frac{\overline{\rho}_{STD}}{\overline{\rho}_{MUT}} + \frac{V_{CV} \cdot (\rho_{CVi} - \rho_{CVf})}{V_{STD} \cdot \overline{\rho}_{MUT}} \right] \tag{17}$$

This can be rewritten as

$$\overline{\dot{V}}_{MUT} = \overline{\dot{V}}_{STD} \cdot \left[\frac{\overline{\rho}_{STD}}{\overline{\rho}_{MUT}} + \frac{V_{CV}}{V_{STD}} \cdot \frac{(\rho_{CVi} - \rho_{CVf})}{\overline{\rho}_{MUT}} \right] \tag{18}$$

or

$$\overline{\dot{V}}_{MUT} = \overline{\dot{V}}_{STD} \cdot \left[\frac{\overline{\rho}_{STD}}{\overline{\rho}_{MUT}} + \frac{V_{CV}}{V_{STD}} \cdot \left(\frac{\rho_{CVi}}{\overline{\rho}_{MUT}} - \frac{\rho_{CVf}}{\overline{\rho}_{MUT}} \right) \right] \tag{19}$$

In a fluid of given thermal expansion coefficient, β, the ratio of density at different temperatures, T_A & T_B can be expressed as

$$\frac{\rho_{T_A}}{\rho_{T_B}} = 1 + \beta \cdot (T_B - T_A) \tag{20}$$

During a period of time, t_c, the ratio of average densities over the period would be determined using the temporal average temperatures.

$$\frac{\overline{\rho}_{STD}}{\overline{\rho}_{MUT}} = 1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right) \tag{21}$$

For determining the initial and final densities of fluid inside the connecting volume, spatial average temperatures at the initial and final times for the standard volume delivery apply. Thus,

$$\frac{\rho_{CVi}}{\overline{\rho}_{MUT}} = 1 + \beta \cdot \left(\overline{T}_{MUT} - \langle T_{CVi} \rangle \right) \tag{22}$$

$$\frac{\rho_{CVf}}{\overline{\rho}_{MUT}} = 1 + \beta \cdot \left(\overline{T}_{MUT} - \langle T_{CVf} \rangle \right) \tag{23}$$

And

$$\left(\frac{\rho_{CVf}}{\overline{\rho}_{MUT}} - \frac{\rho_{CVi}}{\overline{\rho}_{MUT}} \right) = \beta \cdot \left(\langle T_{CVf} \rangle - \langle T_{CVi} \rangle \right) \tag{24}$$

And equation 35 becomes

$$\overline{\dot{V}}_{MUT} = \overline{\dot{V}}_{STD} \cdot \left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD} \right) + \frac{V_{CV}}{V_{STD}} \cdot \beta \cdot \left(\langle T_{CVf} \rangle - \langle T_{CVi} \rangle \right) \right] \tag{25}$$

B. Sensitivity Coefficients

Calibrator Pulses, P_C [Calibrator Pulses]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial P_C} = -P_C^{-1} \tag{1}$$

Time Calibrator, t_C [seconds]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial t_C} = t_C^{-1} \tag{2}$$

Turbine Meter Pulses, P_{MUT} [Meter Pulses]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial P_{MUT}} = P_{MUT}^{-1} \tag{3}$$

Meter Time, t_{MUT} [seconds]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial t_{MUT}} = -t_{MUT}^{-1} \tag{4}$$

Calibrator Constant (Water Draw), K_C [Calibrator Pulses/Unit Volume]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial K_C} = K_C^{-1} \tag{5}$$

Thermal Expansion of Encoder, α_{ENC} [°F^{-1}]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial \alpha_{ENC}} = \frac{-(T_{AMB} - T_{REF})}{\left[1 - \alpha_{ENC} \cdot (T_{AMB} + T_{REF})\right]} \tag{6}$$

Ambient Temperature, T_{AMB} [°F]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial T_{AMB}} = \frac{-\alpha_{ENC}}{\left[1 - \alpha_{ENC} \cdot (T_{AMB} + T_{REF})\right]} \tag{7}$$

Thermal Expansion Coefficient of Flow Tube, α_T [°F^{-1}]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial \alpha_T} = \frac{-(T_{STD} - T_{REF})}{\left[1 - \alpha_T \cdot (T_{STD} - T_{REF})\right]} \tag{8}$$

Average Temperature of Calibrator Fluid Media, $\overline{T}_{STD}$ [°F]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial \overline{T}_{STD}} = -\frac{\alpha_T \cdot \left[V_{STD} + V_{STD} \cdot \beta \cdot \left(\overline{T}_{MUT} - T_{REF}\right) + V_{CV} \cdot \beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)\right] - V_{STD} \cdot \beta}{\left[1 - \alpha_T \cdot \left(\overline{T}_{STD} + T_{REF}\right)\right] \cdot \left[V_{STD} + V_{STD} \cdot \beta \cdot \left(\overline{T}_{MUT} - T_{STD}\right) + V_{CV} \cdot \beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)\right]} \tag{9}$$

Reference Temperature, T_{REF} [°F]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial T_{REF}} = \frac{\alpha_{ENC} \cdot \left[1 - \alpha_T \cdot \left(\overline{T}_{STD} + T_{REF}\right)\right] + \alpha_T \cdot \left[1 - \alpha_{ENC} \cdot (T_{AMB} - T_{REF})\right]}{\left[1 - \alpha_T \cdot \left(\overline{T}_{STD} + T_{REF}\right)\right] \cdot \left[1 - \alpha_{ENC} \cdot (T_{AMB} + T_{REF})\right]} \tag{10}$$

Pressure Coefficient of the Flow Tube, α_P [psi^{-1}]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial \alpha_P} = -\frac{(P_{STD} - P_{REF})}{\left[1 + \alpha_P \cdot (P_{STD} - P_{REF})\right]} \tag{11}$$

Pressure of the Calibrator Fluid, P_{STD} [psig]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial P_{STD}} = \frac{-\alpha_P}{\left[1 - \alpha_P \cdot (P_{STD} + P_{REF})\right]} \tag{12}$$

Reference Pressure, P_{REF} [psig]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial P_{REF}} = \frac{\alpha_P}{\left[1 - \alpha_P \cdot \left(P_{STD} + P_{REF}\right)\right]} \tag{13}$$

Thermal Expansion Coefficient of Fluid, β [°F^{-1}]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial \beta} = -\frac{V_{STD} \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right) + V_{CV} \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)}{V_{STD} \cdot \left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right)\right] + V_{CV} \cdot \beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)} \tag{14}$$

Average Temperature of Fluid in the Meter Under Test, $\overline{T}_{MUT}$ [°F]

$$\frac{1}{K} \cdot \frac{\partial K}{\partial \overline{T}_{MUT}} = \frac{-\beta \cdot V_{STD}}{V_{STD} \cdot \left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right)\right] + V_{CV} \cdot \beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)} \tag{15}$$

Connecting Volume, V_{CV} [in^3]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial V_{CV}} = \frac{\beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)}{-\left[V_{STD} \cdot \left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right)\right] + V_{CV} \cdot \beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)\right]} \tag{16}$$

Volume of Standard, V_{STD} [in^3]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial V_{STD}} = \frac{V_{CV}}{V_{STD}} \cdot \frac{\beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)}{V_{STD} \cdot \left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right)\right] + V_{CV} \cdot \beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)} \tag{17}$$

Connecting Volume Fluid Temperature at the End of Volume Delivery, $\langle t_{CVf} \rangle$ [°F]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial \langle t_{CVf} \rangle} = \frac{-V_{CV} \cdot \beta}{V_{STD}\left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right)\right] + V_{CV} \cdot \beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)} \tag{18}$$

Connecting Volume Fluid Temperature at the Beginning of Volume Delivery, $\langle t_{CVi} \rangle$ [°F]

$$\frac{1}{K_{MUT}} \cdot \frac{\partial K_{MUT}}{\partial \langle t_{CVi} \rangle} = \frac{V_{CV} \cdot \beta}{V_{STD}\left[1 + \beta \cdot \left(\overline{T}_{MUT} - \overline{T}_{STD}\right)\right] + V_{CV} \cdot \beta \cdot \left(\langle t_{CVf} \rangle - \langle t_{CVi} \rangle\right)} \tag{19}$$

C. Fluid Draw Uncertainty for 50 gpm Stand

The following is the uncertainty analysis from a fluid draw performed on the 50 gpm liquid flow stand.

Influence Quantities	Variable	Value	Units	Distribution							
				Normal	Rectangular	One Standard Uncertainty	$\frac{1}{K}\cdot\frac{\partial K}{\partial x}$ Sensitivity Coefficient	Sensitivity Value	Std. Unc (1σ)	Variance (σ^2)	Std Unc. 2σ %
Meniscus Reading Uncertainty	M_E	1.000140	L		0.000105403	6.08546E-05	M_E^{-1}	0.99986002	6.085E-05	3.702E-09	0.01217
Pulses	p	2740.810	[counts]	0.251359676		0.125679838	p^{-1}	0.000364856	4.586E-05	2.103E-09	0.00917
Volume (liters)	V	1.000140	l		0.0003	0.000173205	V^{-1}	0.99986002	1.732E-04	2.999E-08	0.03464
Temperature Calibrator, T_W (°F) (Correlated)	T_W	67.67	°F	0.0144		0.0072	$\frac{\alpha_A[1-\alpha_{7024}\cdot(T_V-T_W)]-\alpha_{7024}\cdot[1+\alpha_A\cdot(T_W-T_S)]}{[1-\alpha_{7024}\cdot(T_V-T_W)]\cdot[1+\alpha_A\cdot(T_W-T_S)]}$	5.267E-04	3.792E-06	1.438E-11	0.00076
Temperature Detector, T_d (°F) (Correlated)	T_d	67.30	°F	0.0144		0.0072	$\frac{\alpha_{ENC}}{1+(T_d-T_{STD})\cdot\alpha_{ENC}}$	4.44E-06	3.1968E-08	1.022E-15	0.00001
Temperature Vessel, T_V (°F) (Correlated)	T_V	68.09	°F	0.018		0.009	$\frac{[1-\alpha_{7024}\cdot(T_V-T_W)][-\alpha_{7024}\cdot(1+\alpha_V\cdot(T_V-T_S))-\alpha_V\cdot(1-\alpha_{7024}\cdot(T_V-T_W))]}{[1+\alpha_V\cdot(T_V-T_S)]}$	5.50E-04	4.9467E-06	2.447E-11	0.00099
Standard Temperature (°F)	T_{STD}	68.00	°F	-	-	-	-	-	-	-	-
Standard Pressure (psig)	P_{STD}	0.0	psi	-	-	-	-	-	-	-	-
Calibrator Pressure, P (psig)	P_{CAL}	10.030	psi	0.006		0.003	$\frac{d+\gamma_{FT}\cdot T\cdot Z_{7024}}{\gamma_{FT}\cdot T(1-P_{CAL}\cdot Z_{7024})\cdot(1+(P_{CAL}\cdot d/\gamma_{FT}\cdot T))}$	8.05E-06	2.4160E-08	5.837E-16	0.00000
Compressibility Factor (MIL-C-7024)	Z_{7024}	6.91E-06	in^2/lbf		6.91E-09	3.98949E-09	$\frac{P_{CAL}}{1-P_{CAL}\cdot Z_{7024}}$	1.0030E+01	4.0016E-08	1.601E-15	0.00001
Volume Thermal Expansion Coefficient of Fluid (7024)	α_{7024}	5.44E-04	°F^{-1}	2.06E-04		0.000103134	$\frac{(T_V-T_W)}{(1+\alpha_{7024}\cdot(T_V-T_W))}$	4.1772E-01	4.3081E-05	1.856E-09	0.00862
Flow Tube Inside Diameter (inches)	d	6.00E+00	in		0.0001	5.7735E-05	$\frac{P_{CAL}}{\gamma_{FT}\cdot T+P_{CAL}\cdot d}$	1.91E-06	1.1030E-10	1.217E-20	0.00000
Flow Tube Wall Thickness (inches)	T	1.88E-01	in		0.0001	5.7735E-05	$\frac{P_{CAL}\cdot d}{T(T\cdot\gamma_{FT}+P_{CAL}\cdot d)}$	6.11E-05	3.5295E-09	1.246E-17	0.00000
Linear Thermal Expansion Coefficient, ad (encoder)	α_{ENC}	4.44E-06	°F^{-1}		5.56E-07	3.21E-07	$\frac{(T_d-T_{STD})}{1+(T_d-T_{STD})\cdot\alpha_{ENC}}$	7.04E-01	2.2573E-07	5.095E-14	0.00005
Area Thermal Expansion Coefficient, bp (flow tube)	α_A	1.766E-05	°F^{-1}		6.244E-07	3.60483E-07	$\frac{(T_W-T_{STD})}{1+(T_W-T_{STD})\cdot\alpha_A}$	3.264E-01	1.1765E-07	1.384E-14	0.00002
Volume Thermal Expansion Coefficient, vm (draw vessel)	α_V	5.56E-06	°F^{-1}		3.89E-07	2.24526E-07	$\frac{(T_V-T_{STD})}{1+(T_V-T_{STD})\cdot\alpha_V}$	9.12E-02	2.0488E-08	4.198E-16	0.00000
Modulus of Elasticity, E (flow tube)	γ_{FT}	2.800E+07	psi	5.600E+05		280000	$\frac{P_{CAL}\cdot d}{\gamma_{FT}(\gamma_{FT}\cdot T+P_{CAL}\cdot d)}$	4.09E-13	1.1462E-07	1.314E-14	0.00002
Repeatability of Water Draw (Type A)	K	2.741E+03	Pulses/l	4.075E-01		0.203747318	K^{-1}	0.000364856	7.4338E-05	5.526E-09	0.01487
									Combined Variance	4.33E-08	
									Combined Uncertainty (k=1)	0.00020798	
									Expanded Uncertainty for Cps (k=2)	**0.04160 %**	

D. Pressure Expansion Coefficient of Flow Tube, α_P [psig^{-1}]

Utilizing equation 30 on page 16 of this paper the following Table was calculated from pressures ranging from 0 to 100 psi in 5 psi steps.

Observed Pressure (psig)	CPS				
0.0	1.000000				
5.0	1.000006				
10.0	1.000011		γ=	2.80E+07	psi
15.0	1.000017		WT=	0.1875	
20.0	1.000023		ID=	6.000	in
25.0	1.000029				
30.0	1.000034				
35.0	1.000040				
40.0	1.000046				
45.0	1.000051				
50.0	1.000057				
55.0	1.000063				
60.0	1.000069				
65.0	1.000074				
70.0	1.000080				
75.0	1.000086				
80.0	1.000091				
85.0	1.000097				
90.0	1.000103				
95.0	1.000109		Pressure Expansion Coefficient		
100.0	1.000114			1.14286E-06	/psi

Calculating the slope from the data in Table the volume expansion coefficient was calculated for a 316 SS to be $1.14286 \times 10^{-6} / psi$ flow tube with a 6.0 in diameter and a 0.1875 wall thickness.

The uncertainty in the above value was then determined from the calibrator pressure, the flow tube inside diameter, the modulus of elasticity, flow tube wall thickness.

Pressure, P *[psig]*

This is the same pressure uncertainty that is calculated above in the fluid pressure, P_{Fluid} section above except there is a different sensitivity coefficient for determining the pressure expansion coefficient of the flow tube. This sensitivity coefficient is given below

$$\text{Sensitivity Coefficient: } \frac{1}{C_{PS}} \cdot \frac{\partial C_{PS}}{\partial P} = \frac{ID}{\gamma \cdot WT + P \cdot ID}$$

Flow Tube Inside Diameter, ID *[in]*

The Flow Tube inside diameter of 6.0 inches was taken from the Flow Technology Test Report for MT50 SN: MT96090156 dated, 11/05/1997. It was assumed that the nominal value of 6.0 inches was measured accurately to within 0.0001 in. Therefore:

Uncertainty Summary:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $\frac{1}{C_{PS}} \cdot \frac{\partial C_{PS}}{\partial ID} = \frac{P}{\gamma \cdot WT - P \cdot ID}$

Distribution: Rectangular

One Standard Uncertainty: $\frac{0.0001 in}{\sqrt{3}} = 5.7735 \times 10^{-5} in$

Modulus of Elasticity, γ ***[psig]***

The API Manual of Petroleum Measurement Standards Chapter 12-Calculation of Petroleum Quantities Table 7 Modulus of Elasticity Discrimination Levels (E) page 17 states that the modulus of elasticity for 316 Stainless Steel is 28,000,000 psi. The uncertainty associated with this value is given by the Materials Metrology and Standards for Structural Performance book on page 157 to be accurate to within ±2%. Therefore:

Uncertainty Summary:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $\frac{1}{C_{PS}} \cdot \frac{\partial C_{PS}}{\partial \gamma} = -\frac{P \cdot ID}{\gamma (\gamma \cdot WT - P \cdot ID)}$

Distribution: Normal

One Standard Uncertainty: $\frac{(28{,}000{,}000 psig) \cdot (2\% / 100\%)}{2} = 280{,}000 psig$

Flow Tube Wall Thickness, WT ***[in]***

The Flow Tube wall thickness of 0.1875 inches was taken from the Flow Technology Test Report for MT50 SN: MT96090156 Dated, 11/05/1997. It was assumed that the nominal value of 0.1875 inches was measured accurately to within 0.0001 in. Therefore:

Uncertainty Summary:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $\frac{1}{C_{PS}} \cdot \frac{\partial C_{PS}}{\partial WT} = -\frac{P \cdot ID}{WT (\gamma \cdot WT - P \cdot ID)}$

Distribution: Rectangular

One Standard Uncertainty: $\frac{0.0001 in}{\sqrt{3}} = 5.7735 \times 10^{-5} in$

The above uncertainties are then combined in Table 3 below to calculate the total uncertainty of the volume pressure coefficient of expansion for a 316 SS tube with a 6.0 in diameter and a 0.1875 wall thickness.

The uncertainties are combined and expanded by root sum squaring the above uncertainties together and multiplying by 2:

$$U_P = 2 \cdot \sqrt{u_P^2 + u_{ID}^2 + u_\gamma^2 + u_{WT}^2} = 0.0006\%$$

			Distribution		One Std	Sensitvity		1 Std Unc.	Var
Variable	Value	Unit	Norm.	Rect.	Unc	Coeff.	Sens.	σ	σ^2
P	60	psig		3.2	1.84752	$\frac{ID}{\gamma \cdot WT - P \cdot ID}$	1.14E-06	2.11E-06	4.46E-12
ID	6	in		0.001	5.77E-04	$\frac{P}{\gamma \cdot WT - P \cdot ID}$	1.14E-05	6.60E-09	4.35E-17
γ	2.80E+07	psig		1400000	8.1E+05	$-\frac{P \cdot ID}{\gamma(\gamma \cdot WT - P \cdot ID)}$	-2.4E-12	-1.98E-06	3.92E-12
WT	0.1875	in		0.001	5.77E-04	$-\frac{P \cdot ID}{WT(\gamma \cdot WT - P \cdot ID)}$	-3.66E-04	-2.11E-07	4.46E-14
						Combined Variance:			8.42E-12
				0.0005804		Combined Uncertianty (k=1):			2.90E-06
						Expanded Uncertainty (k=2):			0.0006%

Combine and Expanded Uncertainty for Flow Tube Pressure Expansion Coefficient

E. Thermal Expansion Coefficient of the Liquid Media, β [°F^{-1}]

This data collected from the Anton Paar DMA 5000 M is shown in the table that follows.

	DMA 5000 M	Vol Exp	
Temp	Density	β	Δ
°F	g/cm^3	$°F^{-1}$	$°F^{-1}$
50.00	0.816897		
59.00	0.813368	4.82083E-04	-7.9854E-06
68.00	0.809837	4.84460E-04	-5.6091E-06
77.00	0.806301	4.87273E-04	-2.7955E-06
86.00	0.802762	4.89837E-04	-2.3210E-07
95.00	0.799219	4.92564E-04	2.4955E-06
104.00	0.795672	4.95319E-04	5.2498E-06
113.00	0.792115	4.98946E-04	8.8768E-06
		Average	Stdev
		4.90069E-04	1.1991E-05

Thermal Expansion Determined from Temperature and Density Reading Taken from Anton Paar DMA 5000 M Density Meter

The data from the DMA 5000 is shown in the two left columns (the Temperature and the corresponding Density). The thermal expansion β of the liquid was then calculated as follows.

The equation for determining volume based on a change in temperature is given below.

$$V_t = V_0 \cdot \left[1 + \beta \cdot \left(t_t - t_0\right)\right]$$

Where:

V_t = Final Volume
V_0 = Initial Volume
β = Thermal Expansion of the liquid
t_t = Final Temperature
t_0 = Initial Temperature

Since density, ρ, is m/V (mass over volume), V is m/ρ (mass over density) and the above equation becomes.

$$\frac{m}{\rho_t} = \frac{m}{\rho_0} \cdot \left[1 + \beta \cdot \left(t_t - t_0\right)\right]$$

Due to the conservation of mass, the mass is the same for the initial and final measurement and the only value that changes is the density due to a change in volume. Therefore,

$$\frac{1}{\rho_t} = \frac{1}{\rho_0} \cdot \left[1 + \beta \cdot \left(t_t - t_0\right)\right]$$

$$\rho_0 = \rho_t \cdot \left[1 + \beta \cdot \left(t_t - t_0\right)\right]$$

$$\rho_t = \frac{\rho_0}{\left[1 + \beta \cdot (t_t - t_0)\right]}$$

Solving for β the following relationship is obtained.

$$\beta = \frac{\left(\frac{\rho_0}{\rho_t} - 1\right)}{(t_t - t_0)}$$

The average of the expansion coefficient over all the measurements was determined to be $4.90069\text{x}10^{-4}$ °F^{-1}.

The uncertainty of this value was determined from the standard deviation calculated above in Table 1 and the temperature accuracy specification of the Anton Paar DMA 5000 M. The accuracy of the Anton Paar 5000 M is given below:

Density ρ accuracy of DMA 5000 is $5\text{x}10^{-6}$ g/cm^3 [12]. Uncertainty of water used for air water adjustment of DMA 5000 is $11\text{x}10^{-6}$ g/cm^3 from SH calibration certificate number 19595 dated 14 May 2009 [10].

Total Accuracy of DMA:

$$U_{DMA} = 2\sqrt{\left(\frac{5\times10^{-6}\,g/cm^3}{2}\right)^2 + \left(\frac{11\times10^{-6}\,g/cm^3}{2}\right)} = 1.2083\times10^{-5}\,g/cm^3$$

Temperature T is measured to an accuracy of 0.018°F.

However, using the volume expansion equation above to calculate thermal expansion, we notice that the same probe and density meter is used to find the initial and final temperatures and densities: this means that the initial temperature and the final temperature uncertainties are correlated and the same is true for the initial density and final density. Therefore, these uncertainties instead of being root sum squared must be simply added together.

Initial Density, ρ_0 [g/cm^3]

The initial density uncertainty was determined by the first measurement using the Anton Paar DMA 5000 M. Using the accuracy of the DMA 5000 M and the accuracy of the dionized water used to characterize it the overall accuracy of the DMA 5000 M is calculated above to be $\pm12\text{x}10^{-6}$ g/cm^3. This uncertainty will be correlated to the final density measurement and must be added to the final density measurement uncertainty; they cannot simply be root sum squared together.

Uncertainty Summary for Initial Density:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $$\frac{1}{\beta}\cdot\frac{\partial\beta}{\partial\rho_0}=\frac{1}{(\rho_0-\rho_t)}$$

Distribution: Rectangular

One Standard Uncertainty: $$\frac{12.083\times10^{-6}\,g/cm^3}{\sqrt{3}}=6.97615\times10^{-6}\,g/cm^3$$

Final Density, ρ_t [g/cm^3]

Uncertainty Summary for Final Density:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $$\frac{1}{\beta}\cdot\frac{\partial\beta}{\partial\rho_t}=\frac{\rho_0}{\rho_t^2\cdot\left(\frac{\rho_0}{\rho_t}-1\right)}$$

Distribution: Rectangular

One Standard Uncertainty: $$\frac{12.083\times10^{-6}\,g/cm^3}{\sqrt{3}}=6.97615\times10^{-6}\,g/cm^3$$

Initial Temperature, t_0 [°F]

The initial temperature uncertainty was determined by the first measurement using the Anton Paar DMA 5000 and according to the manufacture's specification; this measurement was accurate to within ±0.018°F [11]. This uncertainty will be correlated to the final temperature measurement and must be added to the final temperature measurement uncertainty; they cannot simply be root sum squared together.

Uncertainty Summary for Initial Temperature:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $$\frac{1}{\beta}\cdot\frac{\partial\beta}{\partial t_0}=\frac{1}{(t_t-t_0)}$$

Distribution: Rectangular

One Standard Uncertainty: $\frac{0.018°F}{\sqrt{3}} = 0.010392°F$

Final Temperature, t_t ***[°F]***

Uncertainty Summary for Final Temperature:

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: $\frac{1}{\beta} \cdot \frac{\partial \beta}{\partial t_t} = \frac{1}{\rho_t}$

Distribution: Rectangular

One Standard Uncertainty: $\frac{0.018°F}{\sqrt{3}} = 0.010392°F$

Repeatability of the Thermal Expansion Measurement, β [°F^{-1}]

The repeatability of the Thermal Expansion Measurements was calculated from the Standard Deviation of the Thermal Expansion measurements and determined to be 1.1991x10^{-5} °F^{-1} which is already one standard uncertainty.

Uncertainty Summary

Type of Uncertainty: Relative Type B

Sensitivity Coefficient: β^{-1}

Distribution: Normal

One Standard Uncertainty: 1.1991×10^{-5}

Combined and Expanded MIL-PRF-7024E Type II and Gear Oil Blend Thermal Expansion Uncertainty

See the following Table 5 for the total uncertainty for the determination of the calculated thermal expansion of MIL-C-7024 and Gear Oil blend. The total uncertainty is calculated by summing the uncertainties for the temperature and density measurements and root sum squaring them with the repeatability. Therefore:

			Distribution		One Std	Sensitivity	Sense	Std. Unc.
Variable	Value	Unit	Normal	Rect	Unc	Coeff.	Value	1s
ρ_0	0.816897	g/cm^3		1.2E-05	6.9282E-06	$\frac{1}{(\rho_0 - \rho_t)}$	283.3664	1.9632E-03
ρ_t	0.813368	g/cm^3		1.2E-05	6.9282E-06	$\frac{\rho_0}{\rho_t^2 \cdot \left(\frac{\rho_0}{\rho_t} - 1\right)}$	284.5958	1.9717E-03
t_0	50.00	°F		1.8E-02	0.0103923	$\frac{1}{(t_t - t_0)}$	0.111111	0.00115470
t_t	59.00	°F		1.8E-02	0.0103923	$\frac{1}{\rho_t}$	1.229456	0.01277688
Repeat β	4.90E-04	°F^{-1}	2.40E-05		1.1991E-05	β^{-1}	2040.53	2.45E-02
								Combined and Expanded Uncertiany: 5.6860%

Table 5. Combined and Expanded Uncertainty for β.

$$U = 2\sqrt{(0.196322\% + 0.197174\%)^2 + (0.115470\% + 1.277688\%)^2 + (2.446842\%)^2} = 5.6860\%$$

References

[1] England, Wesley B., "An Uncertainty Analysis for a Positive Displacement Liquid Flow Calibrator Using the Water Draw Technique," Measurement Science Conference Proceedings, Mar 2011.

[2] API "Manual of Petroleum Measurement Standards," Chapter 4-Proving Systems, Section 6-Pulse interpolation, 2nd Ed, May 1999, Errata Apr 2007, Reaffirmed May 2008.

[3] Latsko, Jeremy & Winchester, James, "Accounting for the Impact of Thermal Instability in the Liquid Comprising the Connecting Volume of a Piston Displacement Type Volumetric Flow Rate Standard," NCSLI Proceedings, Aug 2010.

[4] Mitutoyo Spec Sheet for Model AT102-500 Code No. 539-119 Linear Scale Unit, Manual No. 4739GB Series N0. 539.

[5] Measurement Computing PCI-CTR10 Specification Sheet Document Revision 1.2 dated June 2006.

[6] Mitutoyo online catalog specifications.

[7] API "Manual of Petroleum Measurement Standards," Chapter 12-"Calculation of Petroleum Quantities," page 16 Table 6.

[8] NIST Engineering Metrology Tool Box (http://emtoolbox.nist.gov/Temperature/Slide14.asp), 2009.

[9] Marlin Corporation online catalog (http://marlinmfg.com/products/sensors/platRest.html), 2012.

[10] API MPM Ch12.2.1-1995 (R2009) Appendix A-6 page 19.

[11] ASME B40.100-2005.01.01 (R2010) "Pressure Gauges and Attachments"

[12] Anton Paar "Instruction Manual DMA 5000 M," Document C76IB01E.fm, Sept 2, 2009.

[13] ANSI/NCSL Z540 "American National Standard for Calibration – Calibration Laboratories and Measuring and Test Equipment General Requirements," ANSI/NCSL Z540.1-1994 (R2002).

[14] ANSI/NCSL Z540 "American National Standard for Calibration- U.S. Guide to the Expression of Uncertainty in Measurement," ANSI/NCSL-Z540-2-1997 (R2002).

[15] Adams, Thomas M., "A2LA Guide for the Estimation of Measurement Uncertainty In Testing," July 2002. A2LA.

[16] ISO/IEC 17025 International Standard, "*General Requirements for the Competence for Testing and Calibration Laboratories*," 2nd Ed., May, 15 2005, Reference Number ISO/IEC 17025:2005(E).

[17] Taylor, Barry N. and Kuyatt, Chris E. NIST "Technical Note 1297 Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results," 1994 Ed.

[18] T.T. Yeh, Jesús Aguilera, and John D. Wright, "Hydrocarbon Liquid Flow Calibration Service, NIST Special Publication 250-1039."